Mario Collura (CM@SISSA [IT]) in . . .

Entanglement and Magic in Quantum Manybody Systems

New insights from matrix product states

MBQM2024 2024 – Abu Dhabi

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- SIMULATING QUANTUM STATES IS IN GENERAL VERY HARD FOR CLASSICAL COMPUTERS.
- IT IS EXPECTED THAT THE EXACT CLASSICAL SIMULATION OF ARBITRARY QUANTUM SYSTEMS IS INEFFICIENT, AS THE RESOURCE OVERHEAD EXPONENTIALLY GROWS WITH THE SIZE OF THE SYSTEM.

QUANTUM

. . . NESS

ENTANGLEMENT

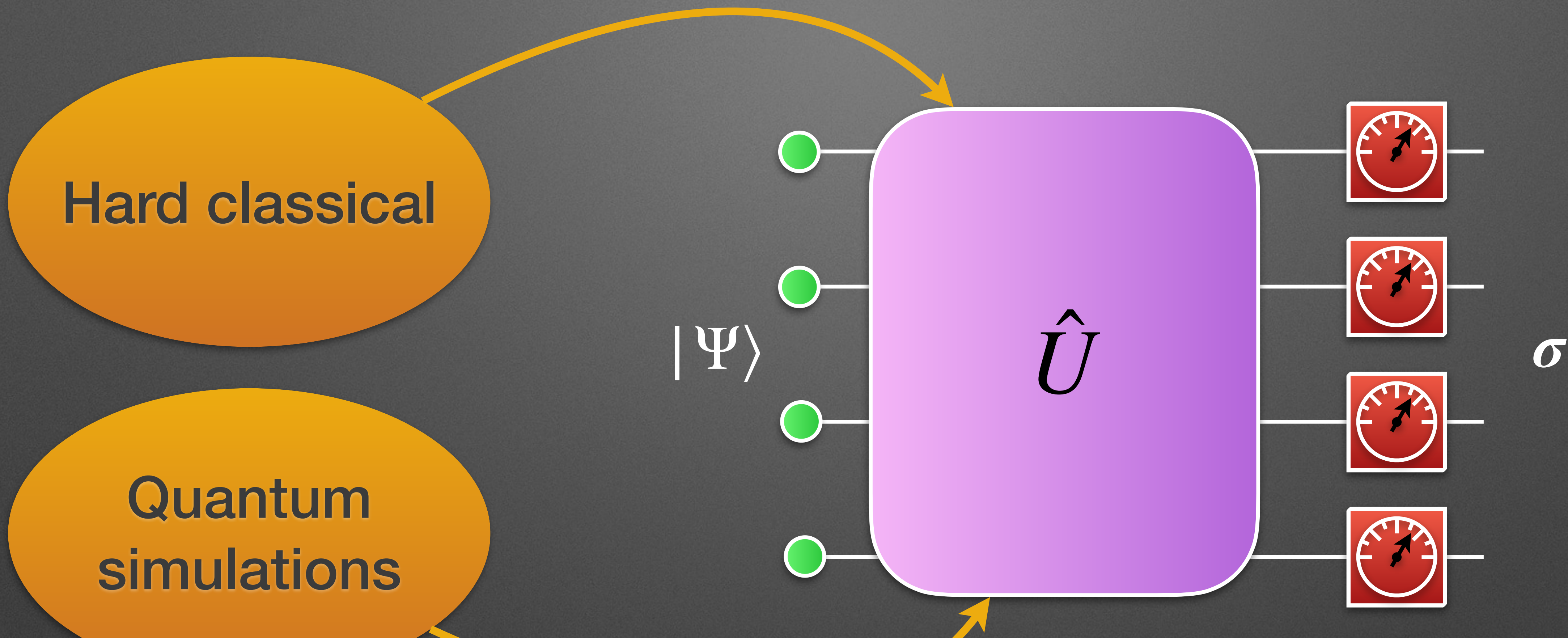
MAGIC

Sergey Bravyi and Alexei Kitaev, Phys. Rev. A 71, 022316 (2005)

QUANTUM ADVANTAGE ONLY WITH BOTH RESOURCES!

ALERT!
LET'S BE CAREFUL

- WE ARE NOT ASKING WHETHER AN OPTIMAL QUANTUM ALGORITHM RUNNING ON AN OPTIMAL QUANTUM COMPUTER WOULD BE **BETTER OR NOT** TO AN OPTIMAL CLASSICAL ALGORITHM RUNNING ON AN OPTIMAL CLASSICAL COMPUTER.
- THE QUESTION IS: <u>DO WE HAVE SITUATIONS WHERE CLASSICAL IS ENOUGH?</u>



$$P(\boldsymbol{\sigma}) = |\langle \boldsymbol{\sigma} | \hat{U} | \Psi \rangle|^2$$

<u>E.G.: DIGITISED QUANTUM ANNEALING</u>

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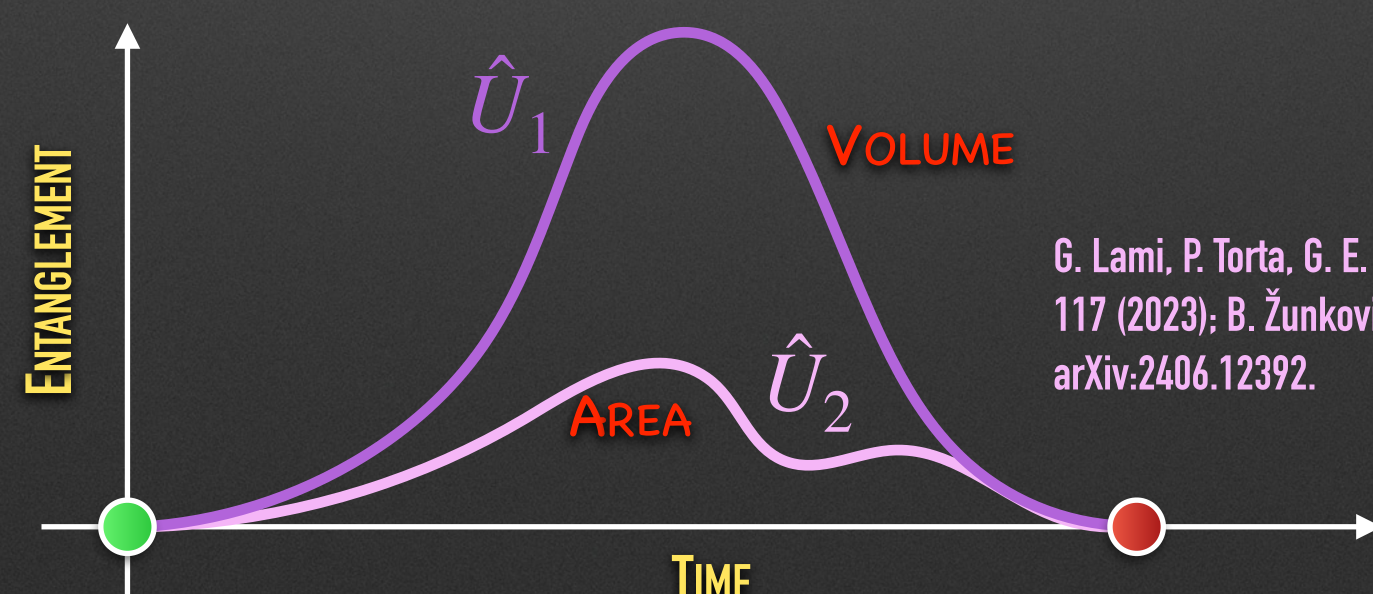
<u>CLASSICAL CONSTRAINED OPTIMISATION PROBLEMS</u>

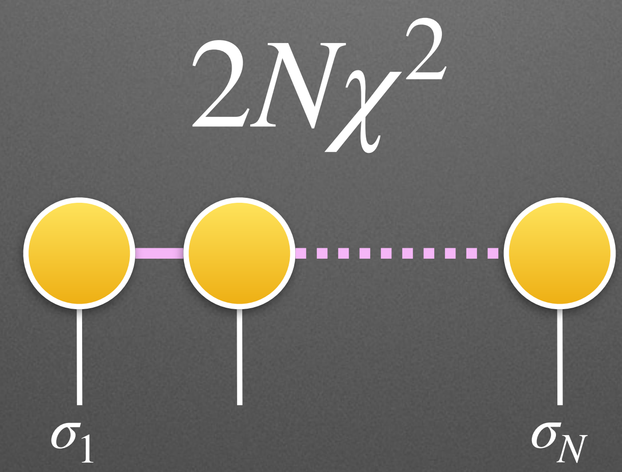


G. Lami, P. Torta, G. E. Santoro, M. Collura, SciPost Phys. 14, 117 (2023); B. Žunkovič, P. Torta, G. Pecci, G. Lami, M. Collura, arXiv:2406.12392.

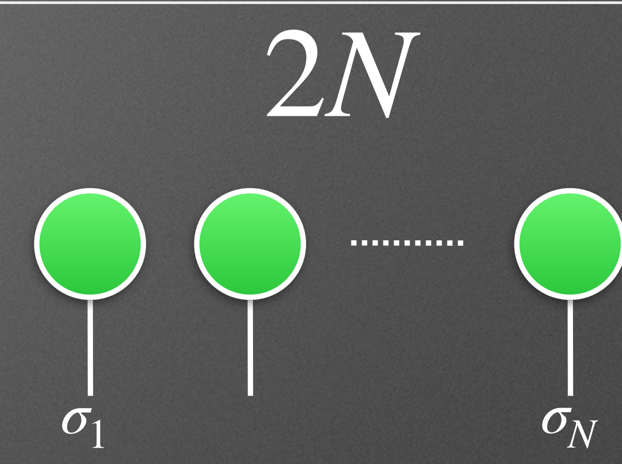
Entanglement

PHYSICISTS AGREE TO IDENTIFY THE ENTANGLEMENT AS A FUNDAMENTAL FEATURE ACCOUNTING FOR QUANTUM COMPLEXITY, THUS MAKING NECESSARY TO EXPLOIT IT PROFICIENTLY IN ANY QUANTUM COMPUTATION.

$$|\Psi\rangle = \Psi(\sigma_1, \ldots \sigma_N)\,|\sigma_1, \ldots \sigma_N\rangle$$



$$\Psi(\sigma_1, \ldots, \sigma_N) = \mathbb{A}_1^{\sigma_1} \cdot \mathbb{A}_2^{\sigma_2} \cdots \mathbb{A}_N^{\sigma_N}$$

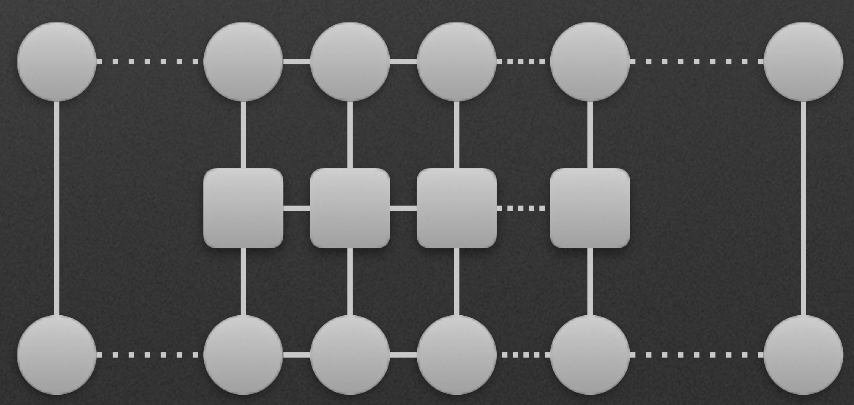


$$\Psi(\sigma_1, \ldots \sigma_N) = \phi_1(\sigma_1) \cdots \phi_N(\sigma_N)$$

Exponentially complex

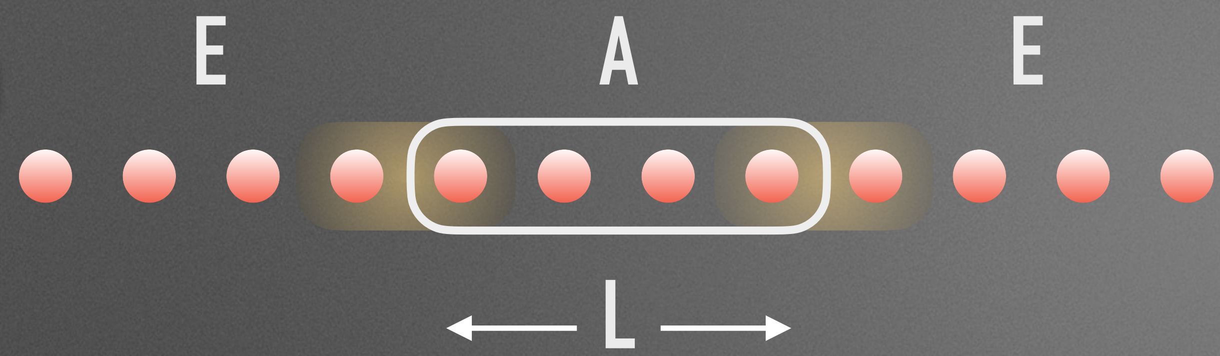
$$\langle \hat{O} \rangle = \sum_{\sigma', \sigma} \phi(\sigma')^* \phi(\sigma)\,\langle \sigma' | \hat{o} | \sigma \rangle$$

Polinomially complex

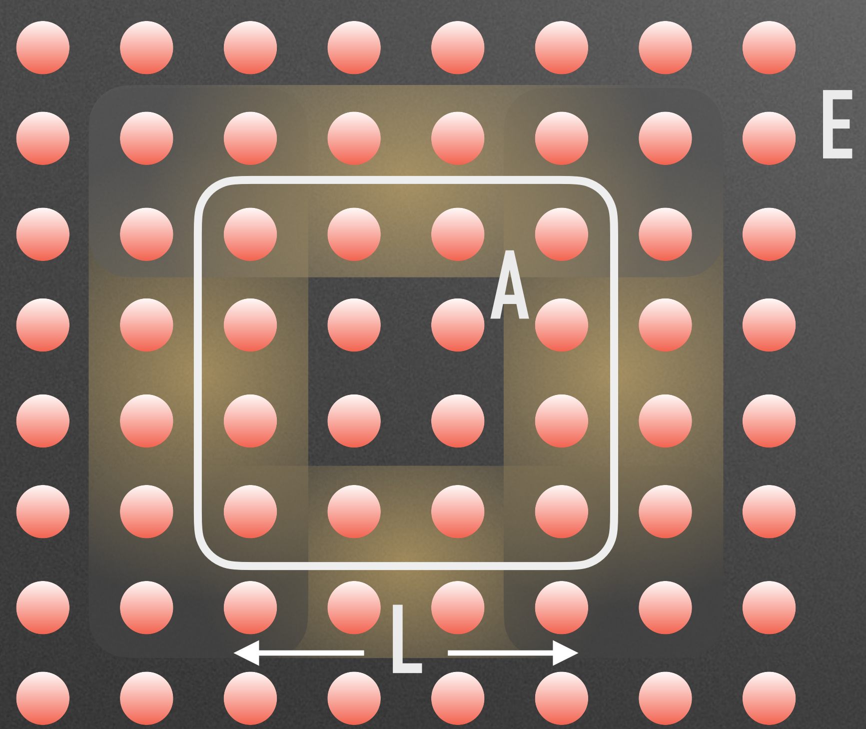


AREA LAW & ENTANGLEMENT ENTROPY

1D



2D



$$\rho_A = \mathrm{Tr}_E |\psi\rangle\langle\psi|$$

$$S(A) = -\,\mathrm{Tr}_A(\rho_A \log \rho_A) = -\sum_j \lambda_j \log \lambda_j$$

RELEVANT EIGENVALUES

$$\chi \approx \exp(S)$$

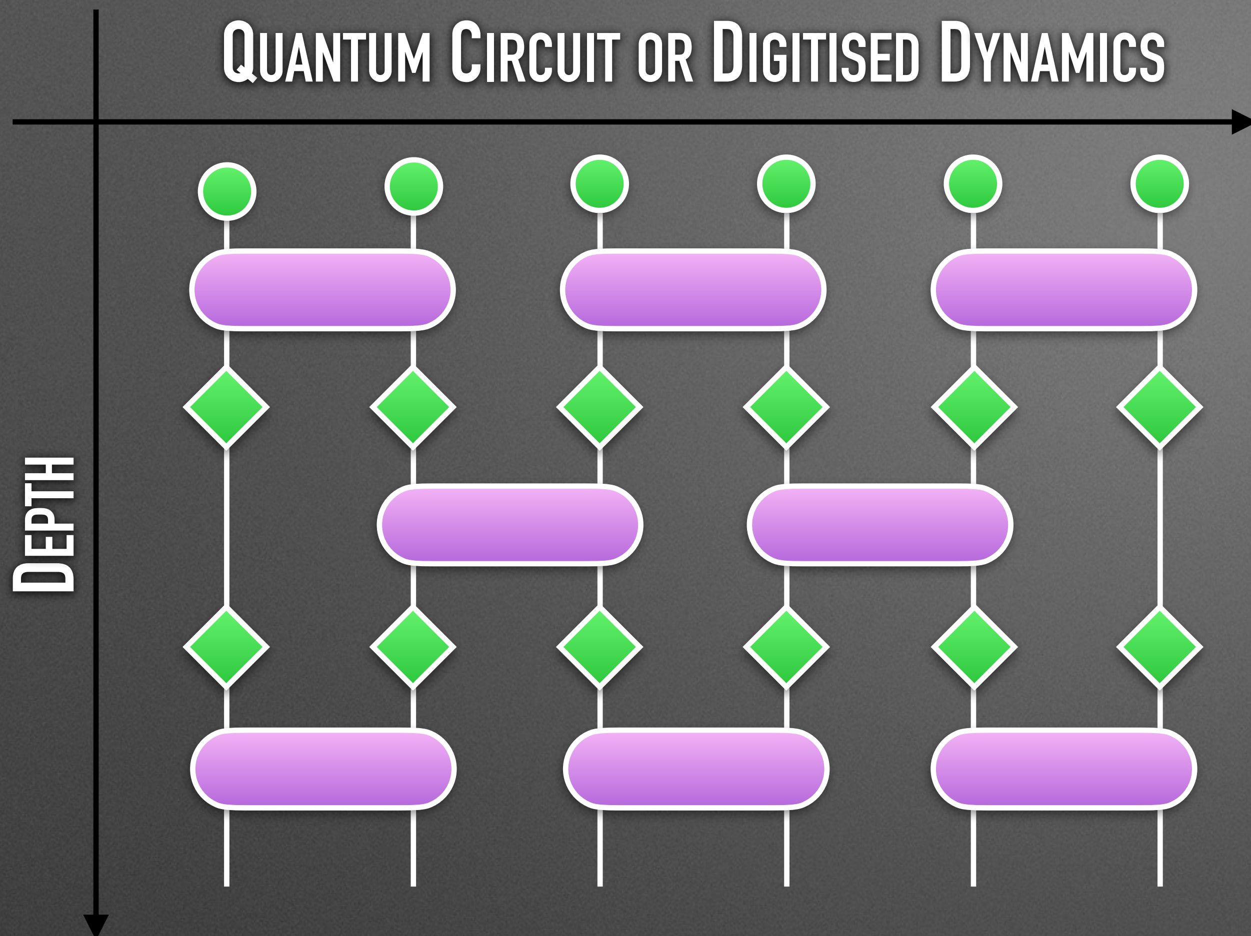
PHYSICAL STATES (LOCAL HAMILTONIANS)

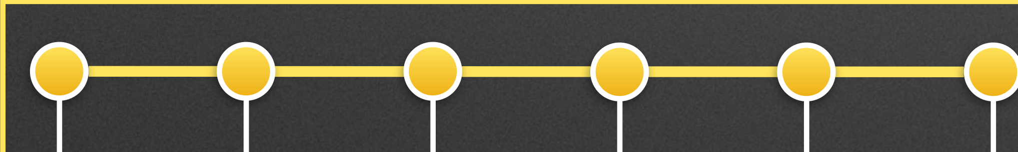
$S(A) \sim L^{d-1}$ (AKA AREA LAW)

1D	$S(A) \sim const$
2D	$S(A) \sim L$

N.B.: SOME CRITICAL GROUND STATES HAVE LOGARITHMIC CORRECTION TO THE AREA LAW

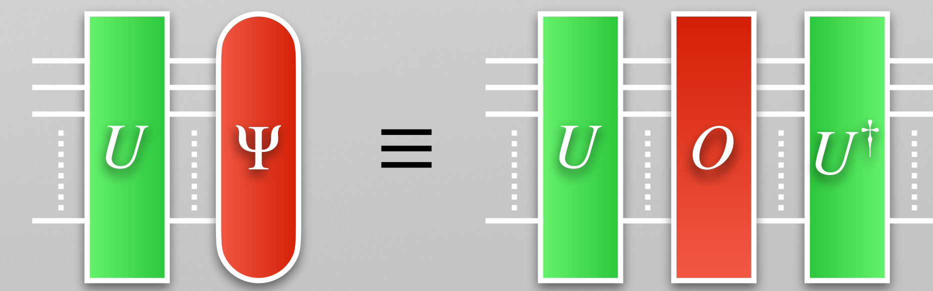
PREPARING AN ENTANGLED STATES IS IN GENERAL AN HARD TASK





MPS with <u>exponentially large bond dimension</u>

Heisenberg Picture



Pauli Group

$$\mathscr{P} = \left\{ e^{i\theta\pi/2} \prod_{j=1}^{N} \sigma_j : \sigma_j \in \{ I, X, Y, Z \} \right\}$$

$\theta = \{0,1,2,3\}$

$$O = \sum_{P \in \mathscr{P}} o(P)\, P$$

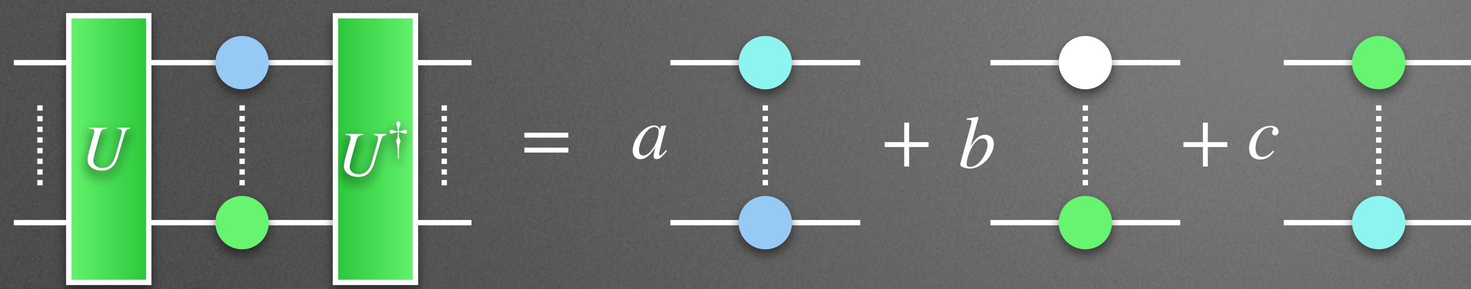
ACTUALLY LESS DUE TO $Y = i\, Z X$

Generating Set (2N)

$\{X_1, \ldots X_N, Z_1, \ldots, Z_N\}$

THE CLIFFORD GROUP

IN GENERAL, UNITARY TRANSFORMATION COULD TAKE A PAULI MATRIX TO ANY OF A RATHER LARGE CLASS OF UNITARY OPERATORS.

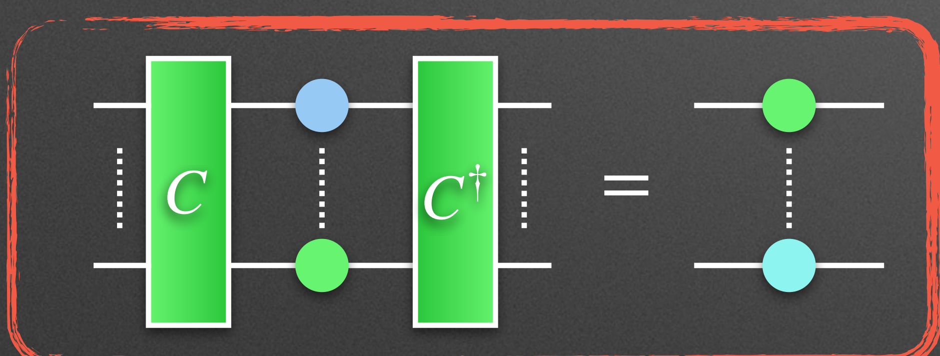


UNITARY GROUP GENERATOR

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \qquad S = \begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$$

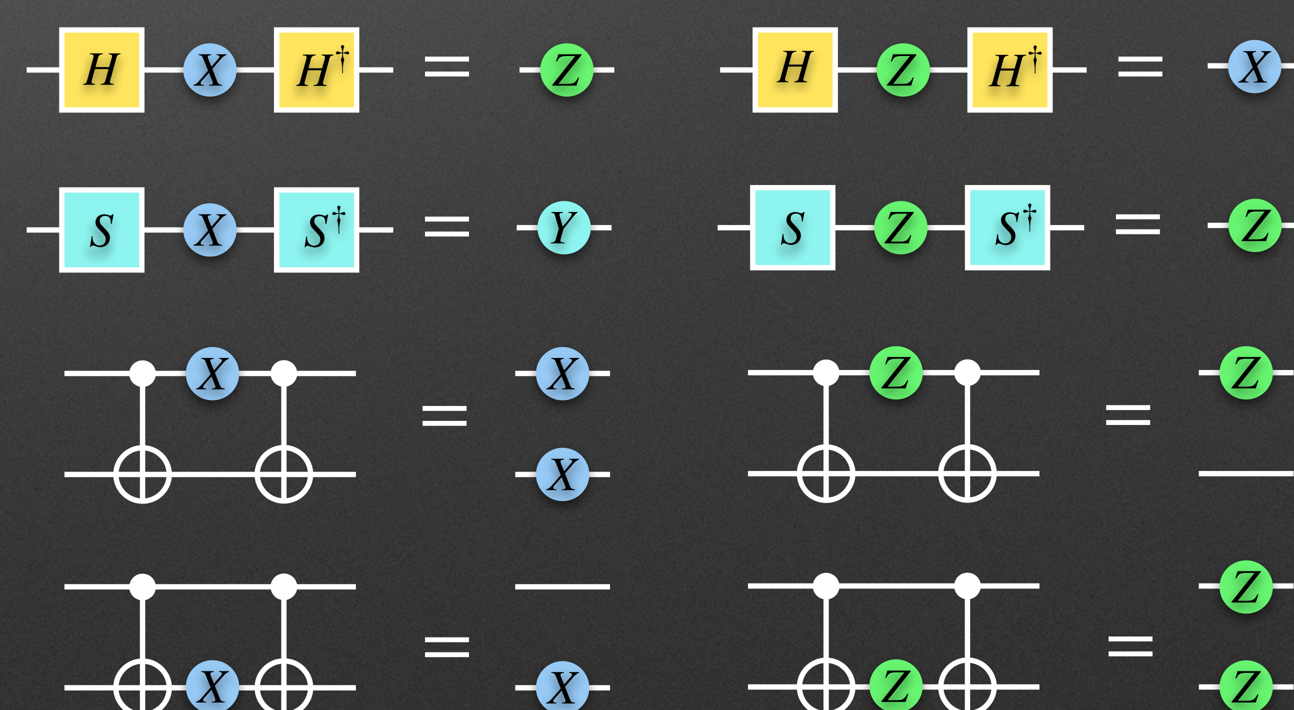
$$\mathrm{CNOT} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix} \qquad T = \begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix}$$

UNIVERSAL QUANTUM GATE SET FOR QUANTUM COMPUTATION.



NORMALISER OF THE PAULI GROUP
IN THE UNITARY GROUP

$$\mathscr{C} = \left\{C \in U_{2^N} : CPC^\dagger = P' \quad for \quad \{P, P'\} \in \mathscr{P}\right\}$$



Daniel Gottesman, "The Heisenberg Representation of Quantum Computers", arXiv:9807006

STABILIZERS

STABILISER GROUP: THE SET OF OPERATORS IN THE PAULI GROUP FOR WHICH THE INPUT STATES ARE +1 EIGENVECTORS. THE SET OF SUCH OPERATORS IS CLOSED UNDER MULTIPLICATION, AND THEREFORE FORMS A GROUP

$$\mathcal{S}(|\psi\rangle) = \left\{ S \in \mathcal{P} : S|\psi\rangle = |\psi\rangle \right\}$$

OR THE OTHER WAY ROUND:
A STATE IS CALLED A STABILISER STATE IF IT IS COMPLETELY DESCRIBED BY SPECIFYING THE STABILISER.

STABILISER GROUP

1. ABELIAN
2. DEGREE 2
3. ISOMORPHIC TO $\mathbb{Z}_2^K, \quad K \leq N$

STABILISER STATE

1. K = N
2. $\#\mathrm{gen}[\mathcal{S}(|\psi\rangle)] = N$
3. $|\psi\rangle$ COMPLETELY SPECIFIED BY THE GENERATORS

WHY SO IMPORTANT?

$$|\psi\rangle = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_{2^N} \end{pmatrix} \qquad \mathbb{S} = \begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_N \end{pmatrix} \Rightarrow \mathbb{M} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN} \end{pmatrix}, \quad \vec{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_N \end{pmatrix}$$

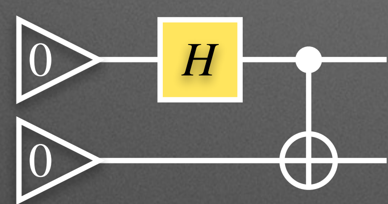
$$\sigma_{ij}, \theta_j \in \{0,1,2,3\}$$

STABILIZERS

THE GOTTESMAN–KNILL THEOREM STATES THAT STABILISER CIRCUITS CAN BE PERFECTLY SIMULATED IN POLYNOMIAL TIME ON A PROBABILISTIC CLASSICAL COMPUTER.

1. PREPARATION OF QUBITS IN COMPUTATIONAL BASIS STATES.
2. CLIFFORD GATES.
3. MEASUREMENTS IN THE COMPUTATIONAL BASIS.

$|00\rangle$



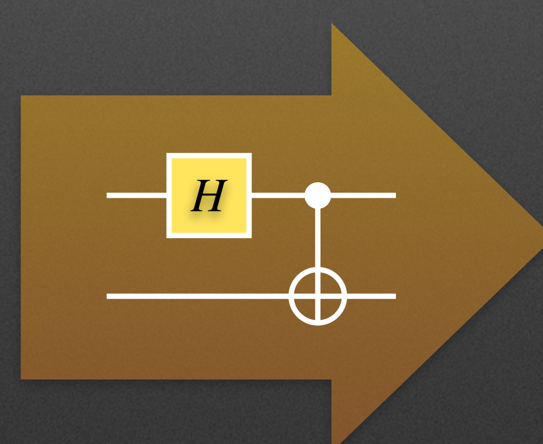
$$= \Big[\ \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$\mathcal{S}$

$$\left\{ \begin{array}{c} I \otimes I \\ I \otimes Z \\ Z \otimes I \\ Z \otimes Z \end{array} \right\}$$

gen$[\mathcal{S}]$

$$\left\{ \begin{array}{c} I \otimes Z \\ Z \otimes I \end{array} \right\}$$



$\mathcal{S}'$

$$\left\{ \begin{array}{c} I \otimes I \\ Z \otimes Z \\ X \otimes X \\ -Y \otimes Y \end{array} \right\}$$

gen$[\mathcal{S}']$

$$\left\{ \begin{array}{c} Z \otimes Z \\ X \otimes X \end{array} \right\}$$

THERE ARE MANY STATES IN THE HILBERT SPACE WHICH ARE NOT STABILISER STATES

COMPUTATIONAL TIME $\sim \mathrm{poly}(N) \times \exp(\# \boxed{T})$

Scott Aaronson and Daniel Gottesman, Phys. Rev. A 70, 052328 (2004).

1. NON-STABILISERNESS (OR MAGIC) IS A FUNDAMENTAL RESOURCE FOR ANY QUANTUM ADVANTAGE

STABILISER NULLITY: $N - \#\mathrm{gen}[\mathcal{S}(|\psi\rangle)]$

NAMELY…HOW MANY "INDEPENDENT" T-GATE WE NEED TO PREPARE A NON-STABILISER STATE

1. <u>HOW TO QUANTIFY MAGIC?</u>

2. <u>HOW TO ENHANCE CLASSICAL SIMULATIONS?</u>

Stabiliser Renyi Entropies

$$M_n(|\psi\rangle) = \frac{1}{1-n} \log \sum_{\sigma \in \mathscr{P}} \frac{1}{2^N} \mathrm{Tr}[\rho\sigma]^{2n} \qquad \rho = |\psi\rangle\langle\psi|$$

L. Leone, S. F. E. Oliviero, and A. Hamma, Phys. Rev. Lett. 128, 050402 (2022).

TO UNDERSTAND THE RELATION WITH USUAL RENYI ENTROPIES

$$\Pi_\rho(\sigma) = \frac{1}{2^N} \mathrm{Tr}[\rho\, \sigma]^2 \geq 0, \qquad \sum_{\sigma \in \mathscr{P}} \Pi_\rho(\sigma) = 1$$

PROBABILITY DISTRIBUTION OVER THE PAULI STRINGS

APART FROM A CONSTANT, DOES COINCIDES WITH THE RÉNYI ENTROPY OF THE DISTRIBUTION

1. $M_n(|\psi\rangle) = 0 \Leftrightarrow |\psi\rangle$ IS A STABILISER, OTHERWISE $M_n(|\psi\rangle) > 0$
2. CONSTANT ON CLIFFORD ORBITS.
3. ADDITIVITY: $M_n(|\psi\rangle \otimes |\phi\rangle) = M_n(|\psi\rangle) + M_n(|\phi\rangle)$

Perfect Pauli Sampling

$$M_n(|\psi\rangle) = \frac{1}{1-n} \log \mathbb{E}[\Pi_\rho(\sigma)^{n-1}] - N \log 2$$

$$M_1(|\psi\rangle) = -\mathbb{E}[\log \Pi_\rho(\sigma)] - N \log 2$$

THE TASK OF SAMPLING FROM THE SET OF THE PAULI STRINGS, WHICH HAS SIZE 4^N, MAY APPEAR AS EXPONENTIALLY HARD.

$$\Pi_\rho(\sigma) = \pi_\rho(\sigma_1)\pi_\rho(\sigma_2 \,|\, \sigma_1)\cdots\pi_\rho(\sigma_N \,|\, \sigma_1 \cdots \sigma_{N-1}) \qquad \pi_\rho(\sigma_j \,|\, \sigma_1 \cdots \sigma_{j-1}) = \frac{\pi_\rho(\sigma_1 \cdots \sigma_j)}{\pi_\rho(\sigma_1 \cdots \sigma_{j-1})}$$

IN OTHER TERMS, THE CONDITIONAL PROBABILITY AT THE STEP j, CAN BE THOUGHT AS THE PROBABILITY $\pi_{\rho_{j-1}}(\sigma_j)$ IN THE PARTIALLY PROJECTED STATE

$$\rho_{j-1} \equiv \frac{\rho\,|_{\sigma_1 \cdots \sigma_{j-1}}}{\pi_\rho(\sigma_1 \cdots \sigma_{j-1})^{1/2}}, \quad \mathrm{Tr}[\rho_{j-1}^2] = 1$$

Perfect Pauli Sampling

$$\rho_{j-1} \equiv \frac{\rho\,|_{\sigma_1\cdots\sigma_{j-1}}}{\pi_\rho(\sigma_1\cdots\sigma_{j-1})^{1/2}}, \quad \mathrm{Tr}[\rho_{j-1}^2] = 1$$

ITERATIVE PROCEDURE

$$\rho_j = \pi_{\rho_{j-1}}(\sigma_j)^{-1/2}\rho_{j-1}\,|_{\sigma_j}$$

WE CAN GENERATE THE OUTCOMES (AND THE PROBABILITIES OF THAT OUTCOMES) BY ITERATING OVER EACH SINGLE QUBITS:

- SAMPLING EACH LOCAL PAULI MATRIX ACCORDING TO THE CONDITIONAL PROBABILITIES.
- ONCE A LOCAL OUTCOME OCCURS, THE STATE IS UPDATED ACCORDINGLY.
- THE ITERATION PROCEEDS UNTIL ALL QUBITS ARE SAMPLED.

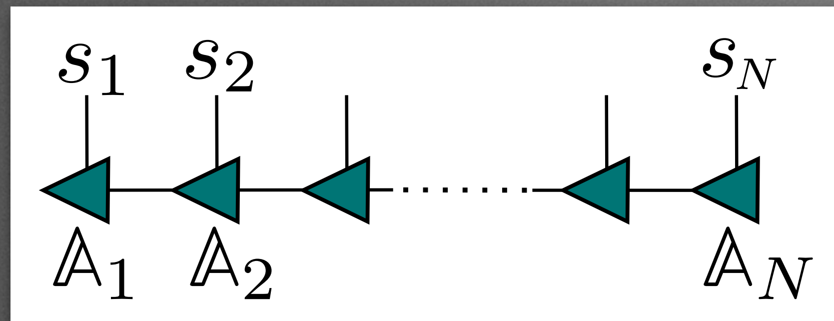
IN ORDER FOR THIS METHOD TO BE COMPUTATIONALLY AFFORDABLE, WE NEED AN EFFICIENT WAY OF:

1. EVALUATING THE CONDITIONAL PROBABILITIES
2. UPDATING THE STATE ACCORDING TO THE LOCAL OUTCOME

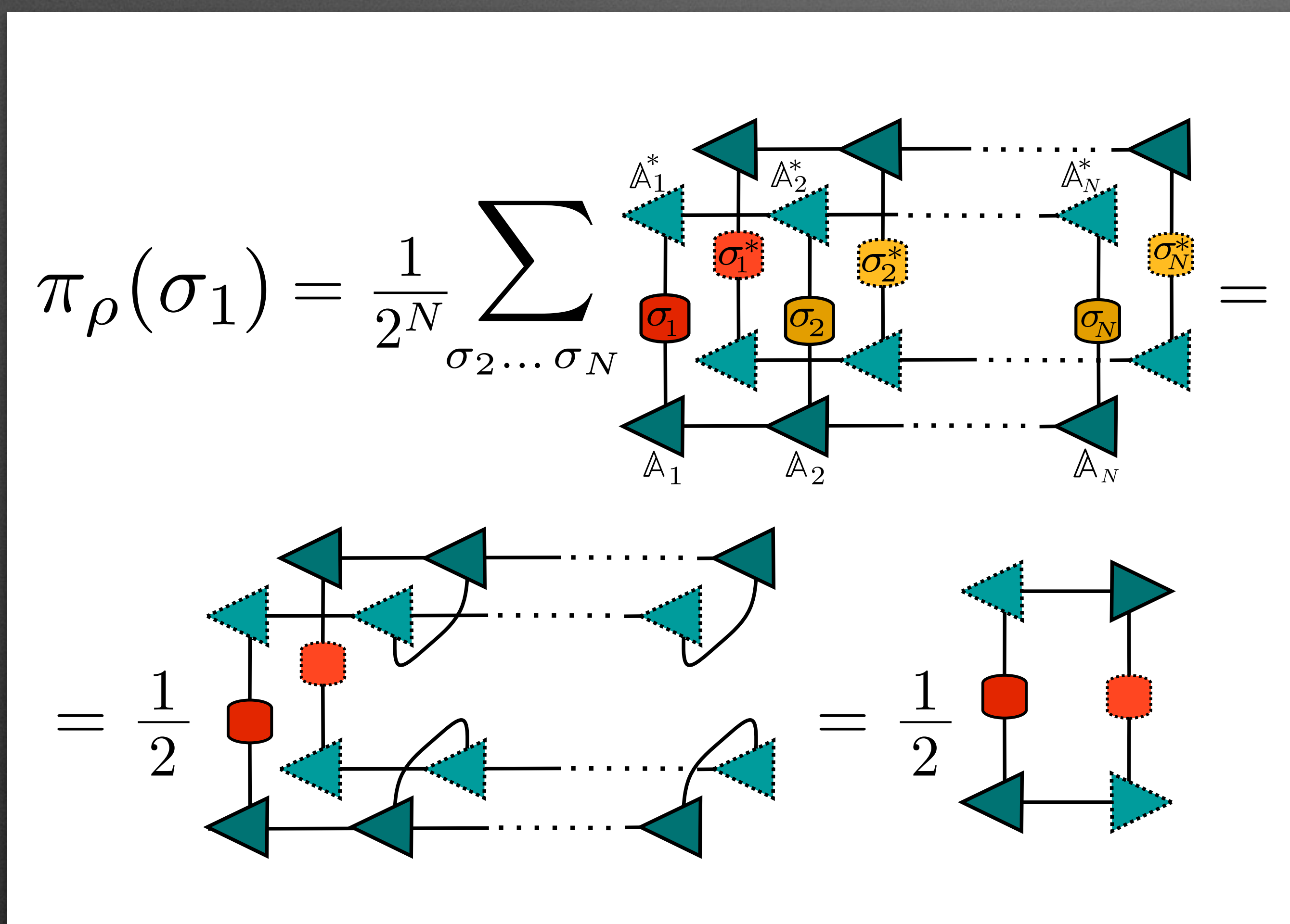
TRUE FOR MPS

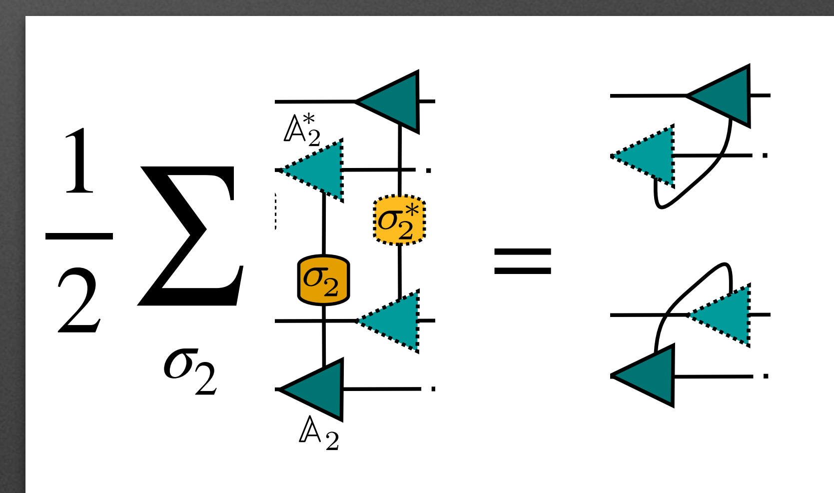
G. Lami and M. Collura, Phys. Rev. Lett. 131, 180401 (2023).

PERFECT PAULI SAMPLING

$|\psi\rangle =$ 

$$\Pi_\rho(\sigma) = \pi_\rho(\sigma_1)\pi_\rho(\sigma_2|\sigma_1)\cdots\pi_\rho(\sigma_N|\sigma_1\cdots\sigma_{N-1})$$





MPS ITERATIVE ALGORITHM

Summary of our Algorithm

- sample $\Pi_\rho(\boldsymbol{\sigma})$ by iteratively sampling the conditional probabilities $\pi_\rho(\sigma_i|\sigma_1 ... \sigma_{i-1})$, for $i = 1, 2, ...N$;
- for an MPS $|\psi\rangle$ of bond dimension χ, the cost of each Pauli sample is $\mathcal{O}(N\chi^3)$;
- the final output is a Pauli string $\boldsymbol{\sigma}$ with probability $\Pi_\rho(\boldsymbol{\sigma})$;
- by repeating $\mathcal{N}$ times, you can find accurate estimations of $M_n(|\psi\rangle)$.

IMPROVED SCALING WITH BOND DIMENSION!!

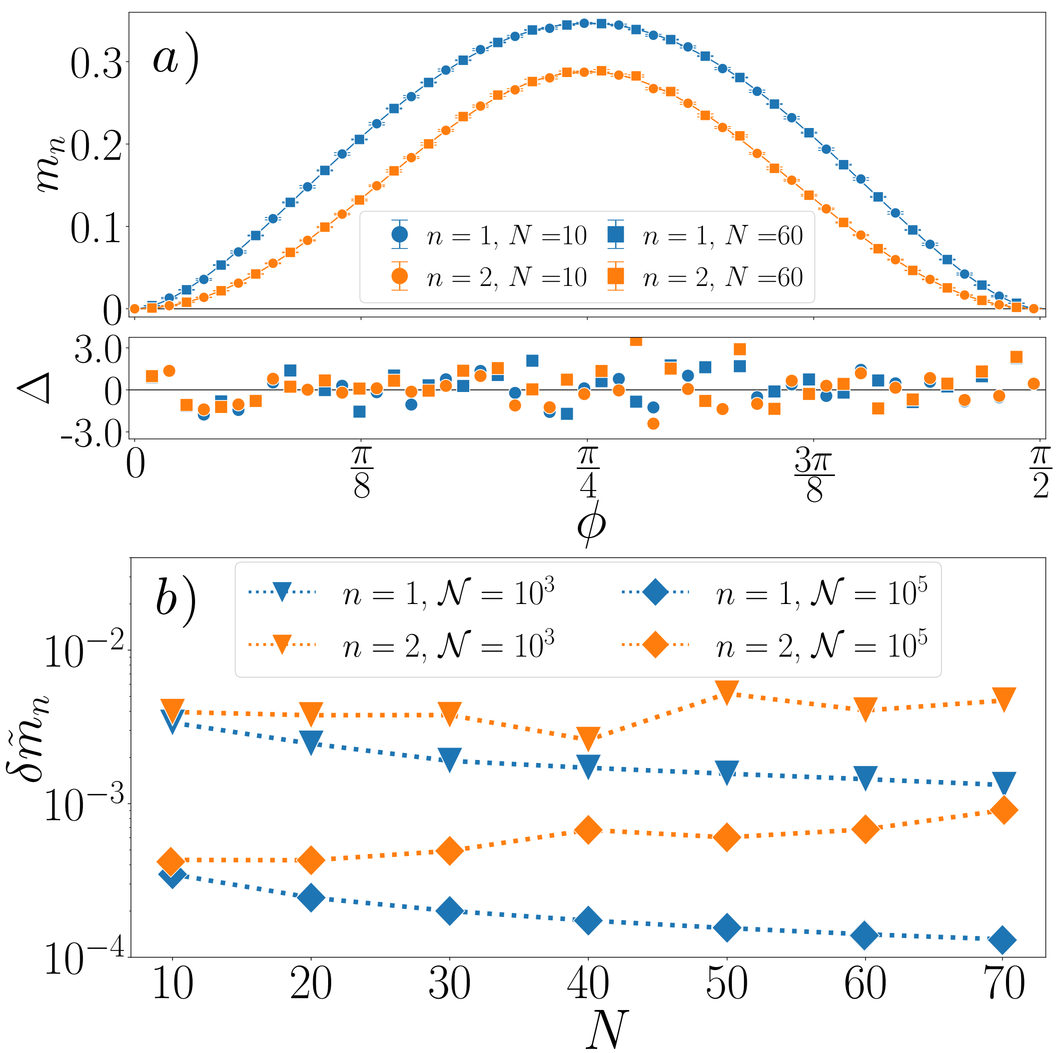
$O(N\chi^{6n})$ → $O(\mathcal{N}N\chi^3)$

T. Haug and L. Piroli, Phys. Rev. B 107, 035148 (2023)

FEW RESULTS

G. Lami and M. Collura, Phys. Rev. Lett. 131, 180401 (2023).

EQUILIBRIUM CHECK

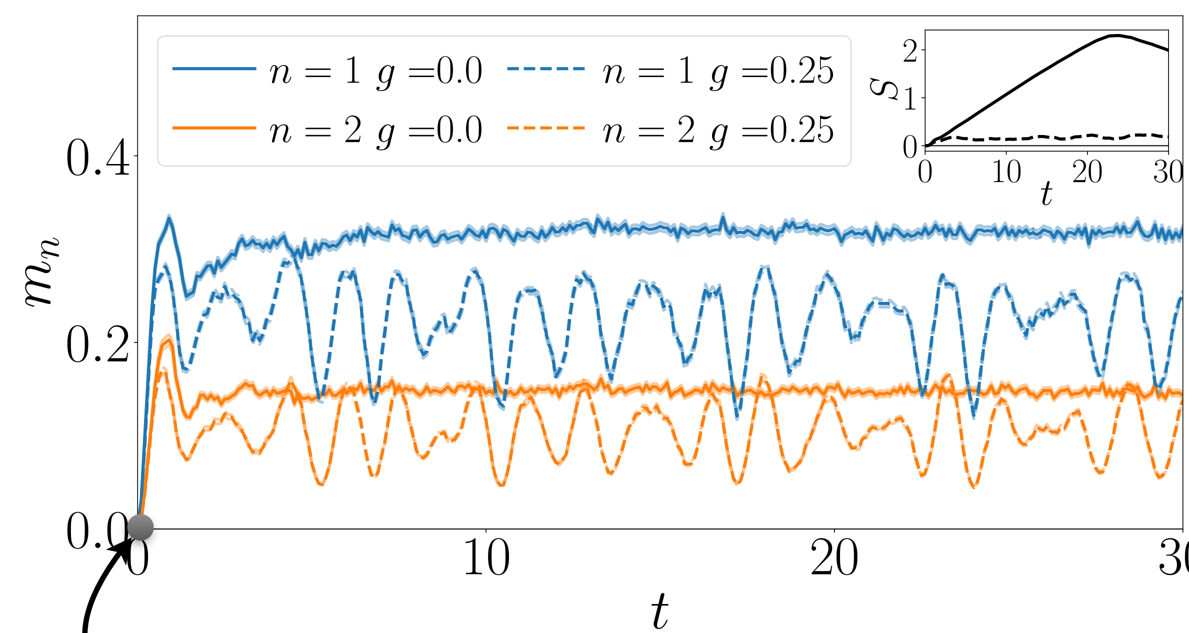


$$\tilde{q}_n = \frac{1}{\mathcal{N}} \sum_{\mu=1}^{\mathcal{N}} \Pi_\rho(\sigma_\mu)^{n-1}$$

$$\tilde{m}_n = (N(1-n))^{-1} \log \tilde{q}_n - \log 2$$

NON EQUILIBRIUM

$$h = 0 \to h = 0.5$$

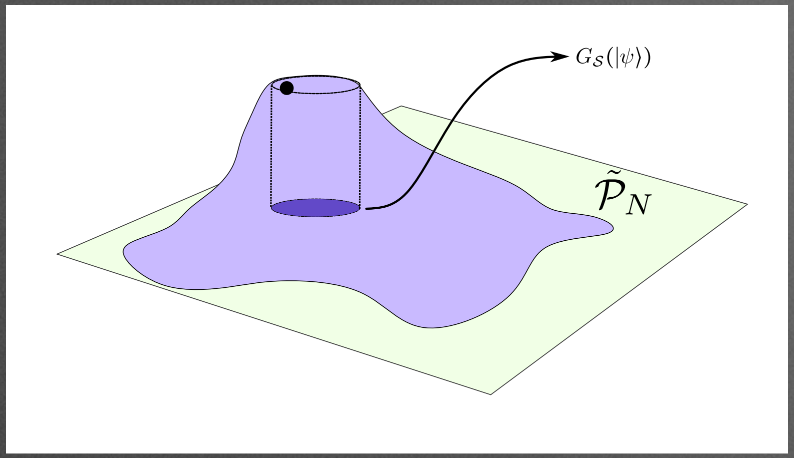


$$|\psi(0)\rangle = |++\ldots+\rangle$$

$$H = -\sum_i \sigma_i^x \sigma_{i+1}^x - h\sum_i \sigma_i^z - g\sum_i \sigma_i^x$$

What about stabiliser group of an MPS ?

CONNECTION TO SREs



THE STABILIZER DIMENSION k_ψ OF $|\psi\rangle$ IS THE NUMBER OF INDEPENDENT (COMMUTING) PAULI STRINGS GENERATING $\mathcal{S}(|\psi\rangle)$ EQUIVALENTLY, ONE CAN DEFINE.

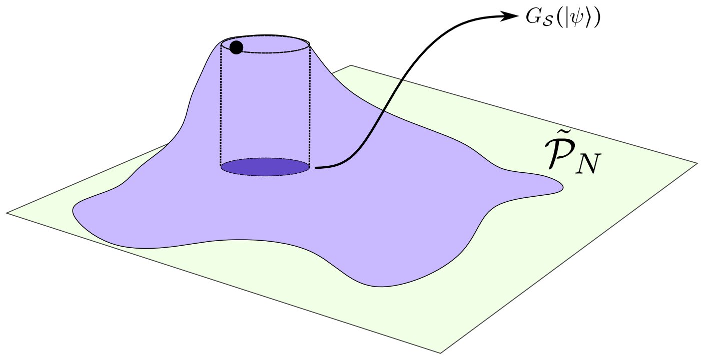
$$\nu = N - \#\mathrm{gen}[\mathcal{S}(|\psi\rangle)]$$

$$M_n(|\psi\rangle) = \frac{1}{1-n} \log \sum_{\sigma \in \mathcal{P}} \frac{1}{2^N} \mathrm{Tr}[\rho\sigma]^{2n} \qquad \rho = |\psi\rangle\langle\psi|$$

$$\lim_{n\to\infty} (n-1)\, M_n(|\psi\rangle) = -\log \sum_{\sigma \in \mathcal{S}(|\psi\rangle)} \frac{1}{2^N} = N \log 2 - \log |\mathcal{S}(|\psi\rangle)|$$

Biasing the Perfect Sampling

Make the sampling biased in order to extract stabilizer strings of the MPS:



This allows for the reconstruction of the entire stabilizer group $G_{\mathcal{S}}(|\psi\rangle)$ associated with the MPS $|\psi\rangle$!

THE STABILIZER DIMENSION k_ψ OF $|\psi\rangle$ IS THE NUMBER OF INDEPENDENT (COMMUTING) PAULI STRINGS GENERATING $\mathcal{S}(|\psi\rangle)$ EQUIVALENTLY, ONE CAN DEFINE.

$$N - \#\mathrm{gen}[\mathcal{S}(|\psi\rangle)]$$

STABILISER STRING PROBABILITY

$$\boxed{\sigma \in \mathcal{S}(|\Psi\rangle) \iff \Pi_\rho(\sigma) = 1/2^N}$$

$$\Pi_\rho(\sigma) = \pi_\rho(\sigma_1)\pi_\rho(\sigma_2|\sigma_1)\cdots\pi_\rho(\sigma_N|\sigma_1\cdots\sigma_{N-1})$$

AT THE GENERIC STEP OF THE SWEEP WE KEEP A CERTAIN NUMBER K OF SUB-STRINGS $\{\sigma^\mu_{[1,i]}\}_{\mu=1}^K$

AND THE CORRESPONDING PARTIAL PROBABILITIES

$$\pi_\rho(\sigma_1\ldots\sigma_i) = \sum_{\vec{\sigma}\in P_{N-i}} \frac{1}{2^N}\mathrm{Tr}[\rho\sigma_1\ldots\sigma_i\vec{\sigma}]^2$$

IDEALLY WE WOULD LIKE TO KEEP TRACK OF ALL POSSIBLE SUB-STRINGS IN ORDER TO IDENTIFY THOSE THAT MEET THE STABILISER CONDITION

IN PRACTICE, ONE HAS TO FIND EFFECTIVE WAYS OF DISCARDING CERTAIN SUB-STRINGS TO ENSURE THAT THEIR TOTAL NUMBER REMAINS WITHIN A PREDEFINED MAXIMUM NUMBER $\mathcal{N}$

HERE THE STRATEGY:

1. FOR ANY STABILISER STRING $\sigma \in \mathcal{S}(|\psi\rangle)$ THE PARTIAL PROBABILITY AT SITE i IS LOWER BOUNDED:

$$\pi_\rho(\sigma_{[1,i]}) \geq 1/(2^i \chi_i)$$

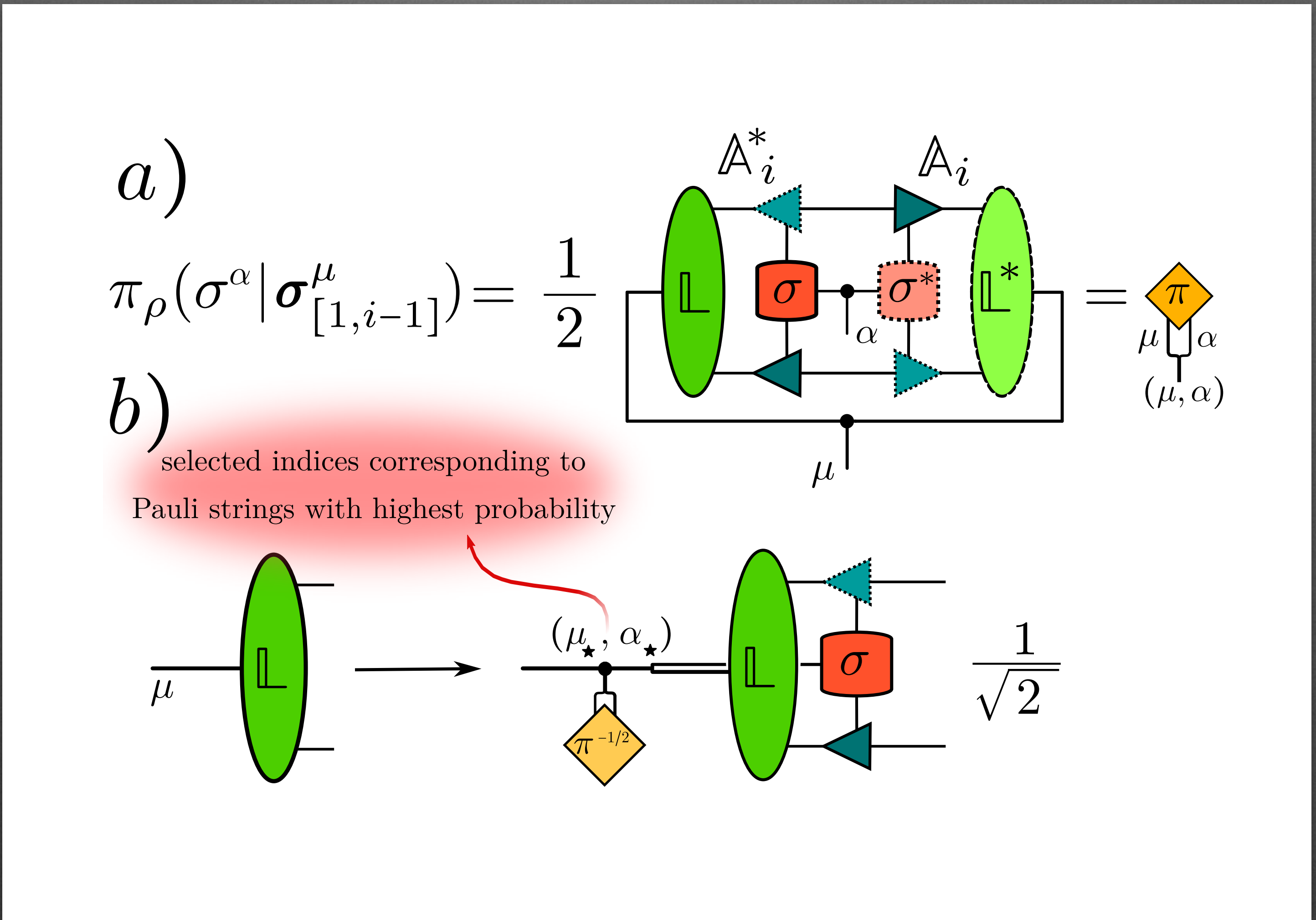
ACCORDING, ONE CAN DISCARD ALL STORED SUB-STRINGS VIOLATING THAT CONDITION.

2. WHEN K EXCEEDS $\mathcal{N}$, ONE CAN SIMPLY SORT THE PARTIAL PROBABILITIES IN DESCENDING ORDER AND SELECT THE SUB-STRINGS CORRESPONDING TO THE HIGHEST $\mathcal{N}$ VALUES.

These are the sub-strings with the highest likelihood to maximise the final probability

G. Lami and M. Collura, Phys. Rev. Lett. 133, 010602 (2024)

GENERIC STEP



$\alpha \in \{0,1,2,3\}$

$\mu \in \{0,\ldots,K\}$

INITIALLY K=1 AND $\mathbb{L}^\mu = (1)$

The output of the algorithm is a set of
$K \leq \mathcal{N}$ stabiliser strings

ITERATION OVER MODIFIED STATE

WHILE THESE STRATEGIES ARE ALREADY EFFECTIVE IN ENSURING A FAVORABLE PROBABILITY OF SAMPLING $\mathcal{S}(|\psi\rangle)$, THIS CAN BE FURTHER INCREASED.

1. REVERSE THE SWEEP
2. REPETE THE SAMPLING ON $|\psi'\rangle = U_C|\psi\rangle$

Reshuffle the partial probabilities

MAP BACK THE SAMPLED PAULI STRINGS

$$\mathcal{S}(|\psi\rangle) = U_C\mathcal{S}(|\psi'\rangle)U_C^\dagger$$

BENCHMARK

$$|N, N_T\rangle \equiv |0\rangle^{\otimes(N-N_T)}|T\rangle^{\otimes N_T}$$

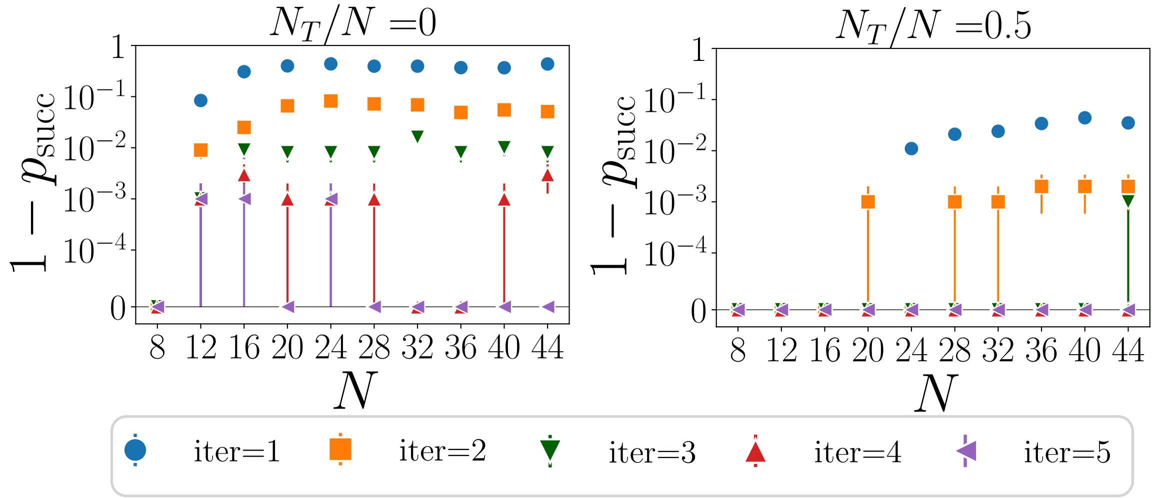
$$|T\rangle = TH|0\rangle = (|0\rangle + e^{i\pi/4}|1\rangle)/\sqrt{2}$$

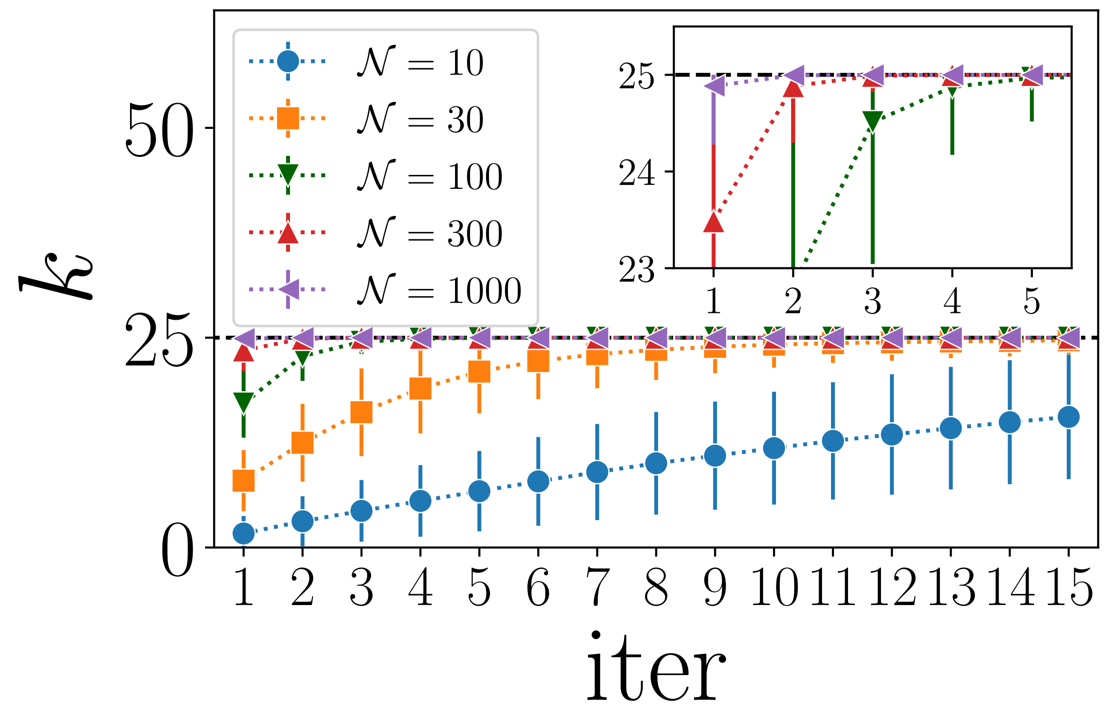
$$|\psi\rangle = U_C|N, N_T\rangle$$

BY CONSTRUCTION, $|\psi\rangle$ HAS STABILIZER DIMENSION $k_\psi = N - N_T$ AND ITS GENERATORS CAN BE OBTAINED BY EVOLVING THE GENERATORS $\{\sigma_1^3 \ldots \sigma_{N-N_T}^3\}$ OF $\mathcal{S}(|N, N_T\rangle)$ IN THE TABLEAU FORMALISM.

$$\mathcal{N} = \mathcal{O}(N)$$

To get all stabiliser string generators in $\mathcal{O}(1)$ iterations





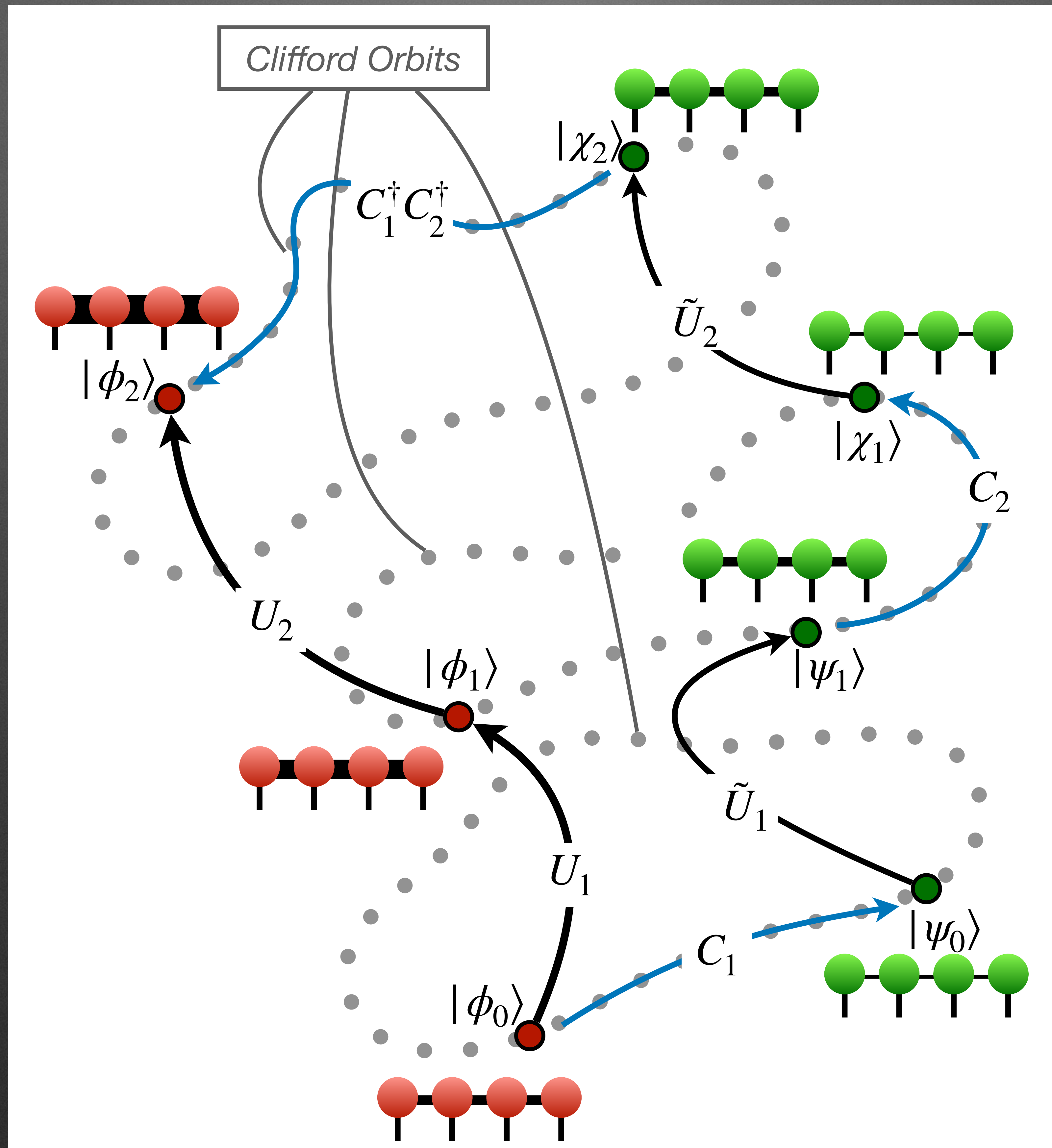
OVER 10^3 DIFFERENT REALISATION OF U_C

PARTIAL SUMMARY

- QUANTUMNESS $\longrightarrow$ ENTANGLEMENT & MAGIC (or NON-STABILISERNESS)
- MAGIC IS IN GENERAL EXPONENTIALLY HARD TO COMPUTE
- VIA PERFECT PAULI SAMPLING $\longrightarrow$ VERY EFFICIENT FOR MPS (e.g.: allow to explore the non-equilibrium!!)
- BIASED PERFECT SAMPLING $\longrightarrow$ EXTRACT STABILISER GROUP OF AN MPS

$$\boxed{O(N^2\chi^3)}$$

WHAT ABOUT ENHANCING THE CLASSICAL SIMULATIONS?



Evolve $|\psi\rangle = \hat{C}_3\hat{R}_2\hat{C}_2\hat{R}_1\hat{C}_1|\psi_0\rangle$ VOLUME-LAW ENTANGLEMENT!!

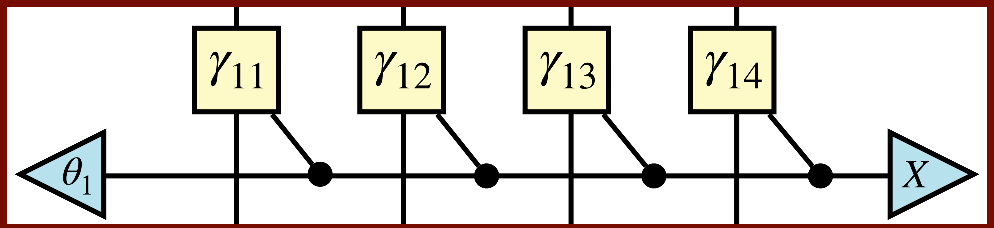
Measure $\hat{O} = \hat{I}\hat{Z}\hat{I}\hat{Z}$

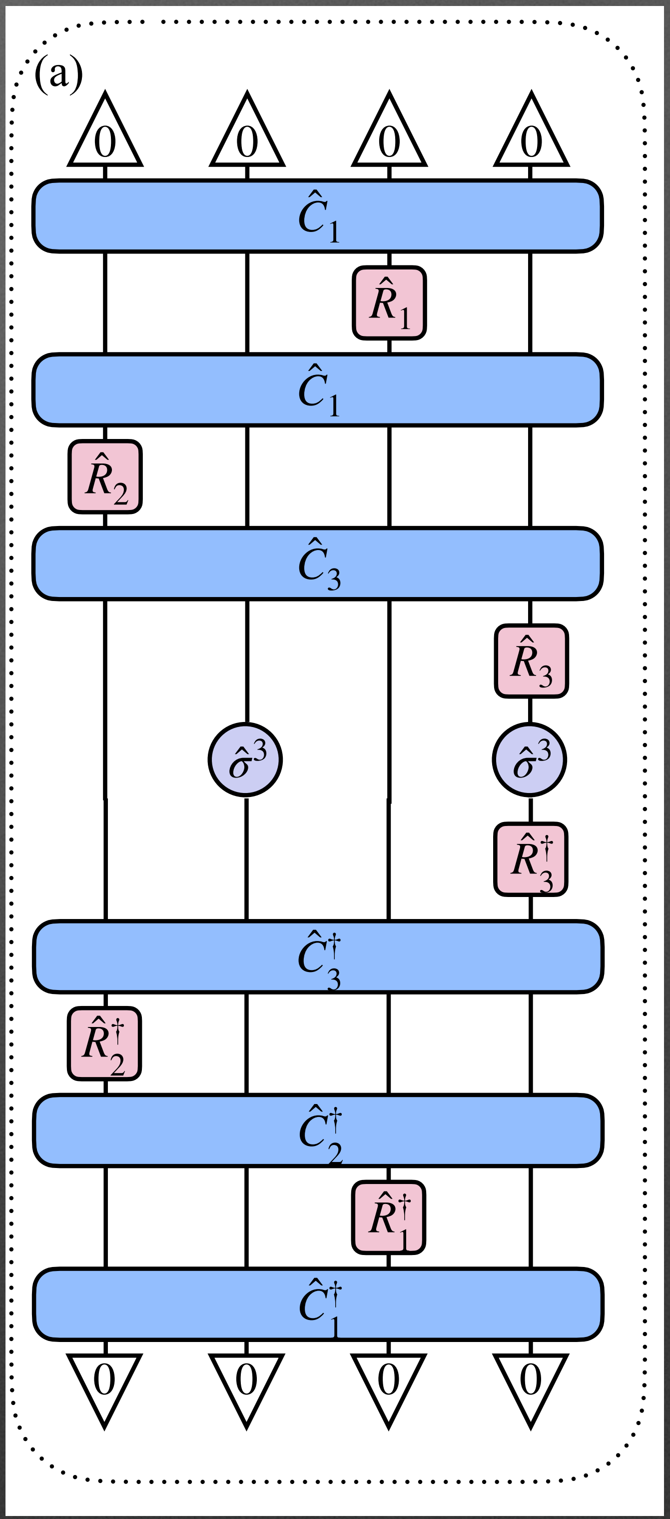
$$\hat{R}_1 = \hat{I}\hat{I}[\cos(\theta_1)\hat{I} - i\sin(\theta_1)\hat{P}]\hat{I}$$
$$= \cos(\theta_1)\hat{I}\hat{I}\hat{I}\hat{I} - i\sin(\theta_1)\hat{I}\hat{I}\hat{P}\hat{I}$$

LETS APPLY THE STABILISER FORMALISM

$$\hat{R}_1\hat{C}_1 = \hat{C}_1\hat{\mathbb{T}}_1$$
$$\hat{\mathbb{T}}_1 = \cos(\theta_1)\hat{I}\hat{I}\hat{I}\hat{I} - i\sin(\theta_1)\hat{\sigma}^{\gamma_{11}}\hat{\sigma}^{\gamma_{12}}\hat{\sigma}^{\gamma_{13}}\hat{\sigma}^{\gamma_{14}}$$

MPO with $\chi = 2$ no matter de Clifford circuit!!!!





Hybrid Stabilizer Matrix Product Operator

$$|\psi\rangle = \hat{C}_3 \hat{R}_2 \hat{C}_2 \hat{C}_1 \hat{\mathbb{T}}_1 |\psi_0\rangle$$

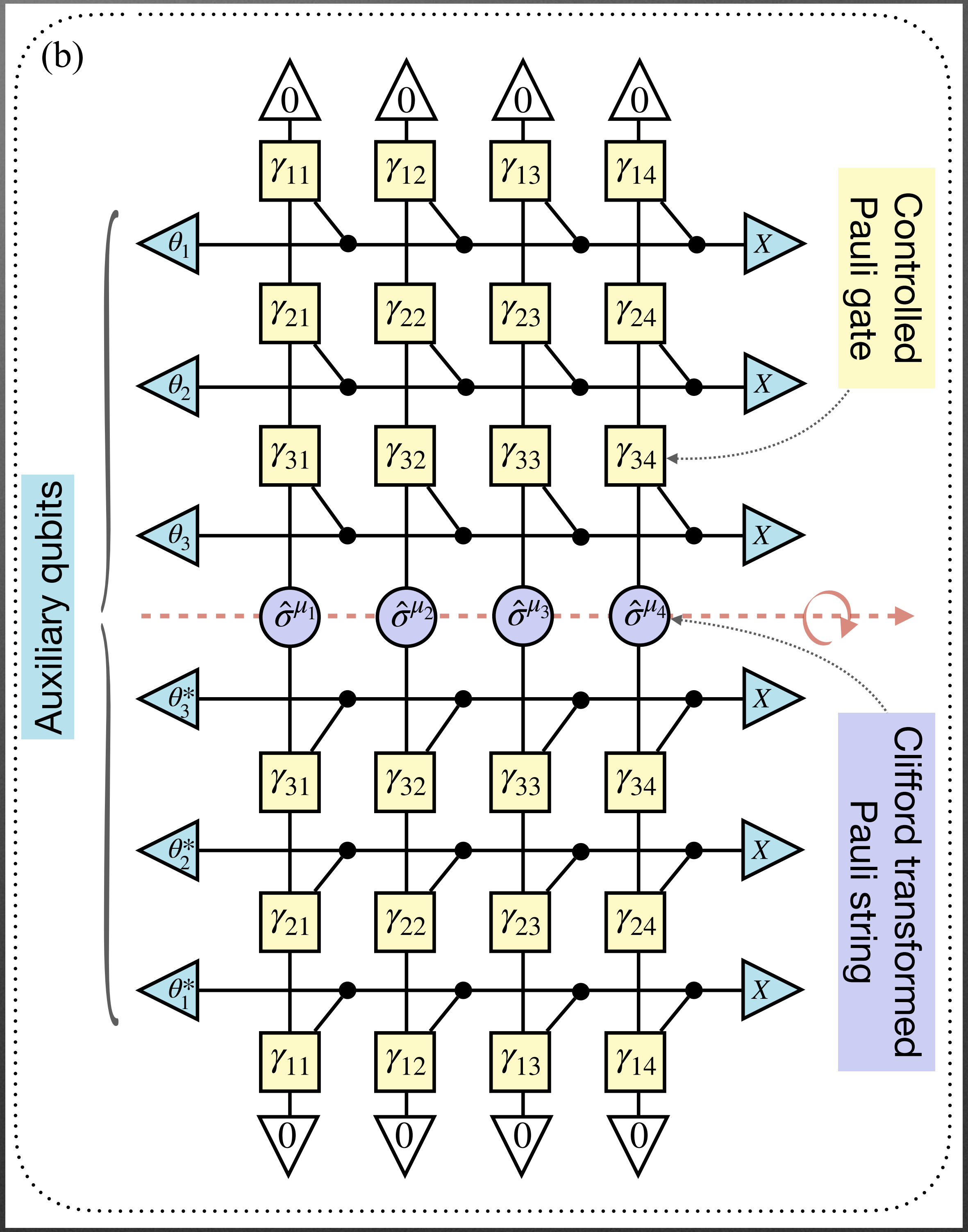
$$|\psi\rangle = \boxed{\hat{C}_3 \hat{C}_2 \hat{C}_1} \hat{\mathbb{T}}_2 \hat{\mathbb{T}}_1 |\psi_0\rangle$$

EASILY TRANSFORM THE OBSERVABLE

$$\hat{O} = \hat{I}\hat{Z}\hat{I}\hat{Z} \Rightarrow \hat{O}' = \hat{\sigma}^{\mu_1}\hat{\sigma}^{\mu_2}\hat{\sigma}^{\mu_3}\hat{\sigma}^{\mu_4}$$

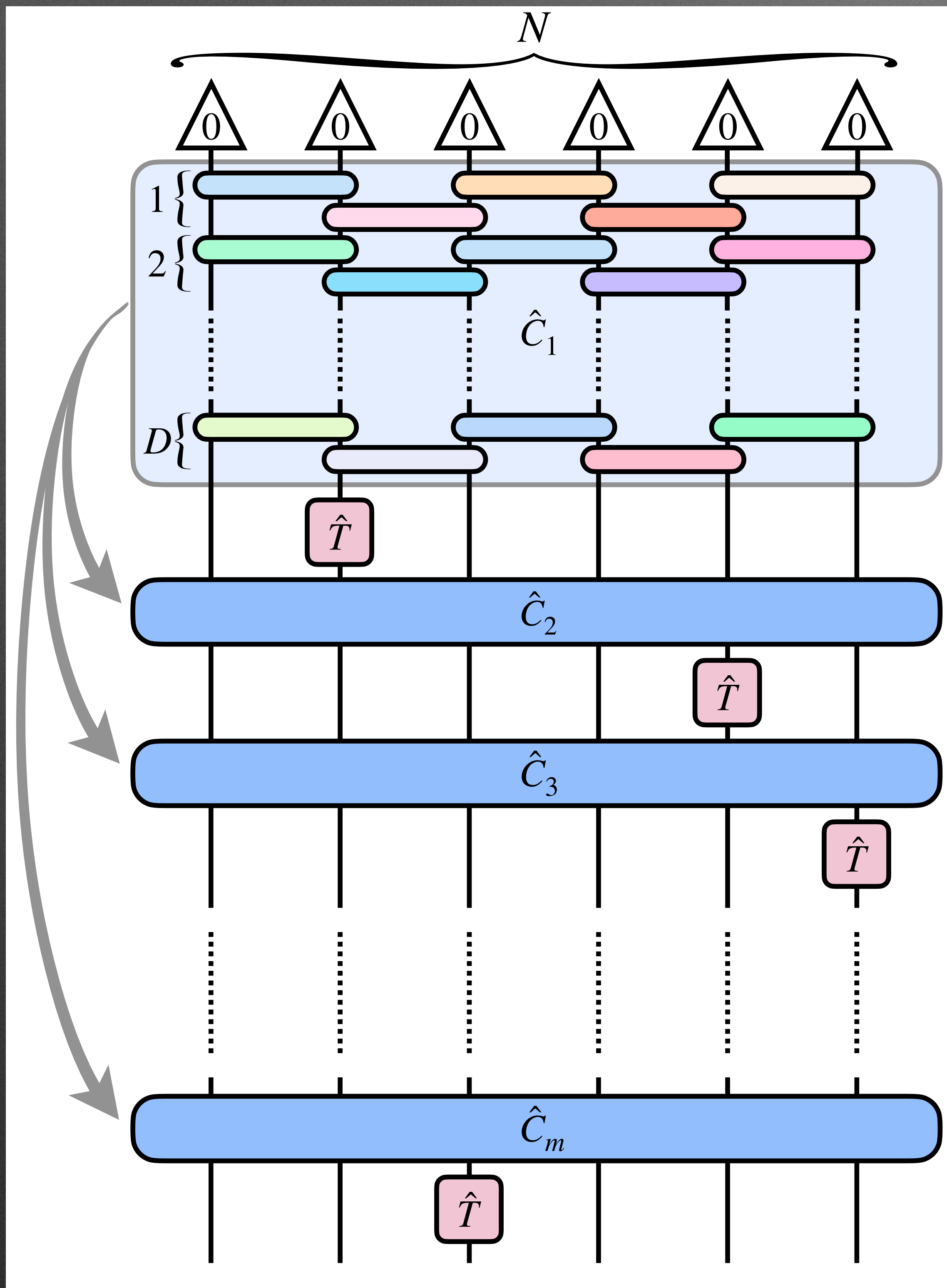
Compute

$$\langle \psi_0 | \hat{\mathbb{T}}_1^\dagger \hat{\mathbb{T}}_2^\dagger \hat{O}' \hat{\mathbb{T}}_2 \hat{\mathbb{T}}_1 | \psi_0 \rangle$$



AF Mello, A Santini, M Collura, arXiv:2405.06045 (accepted on PRL)

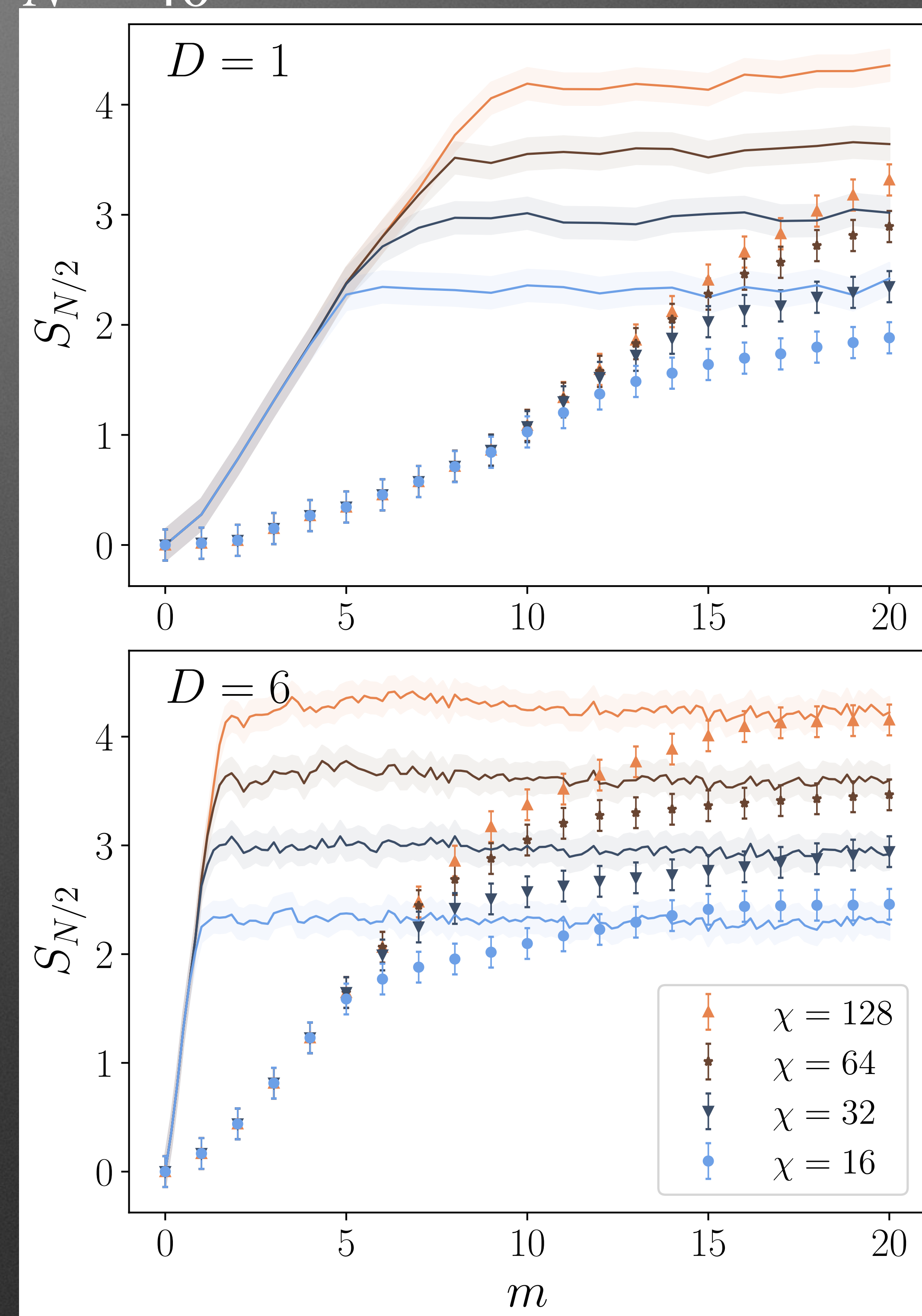
Random Clifford T-doped Circuit



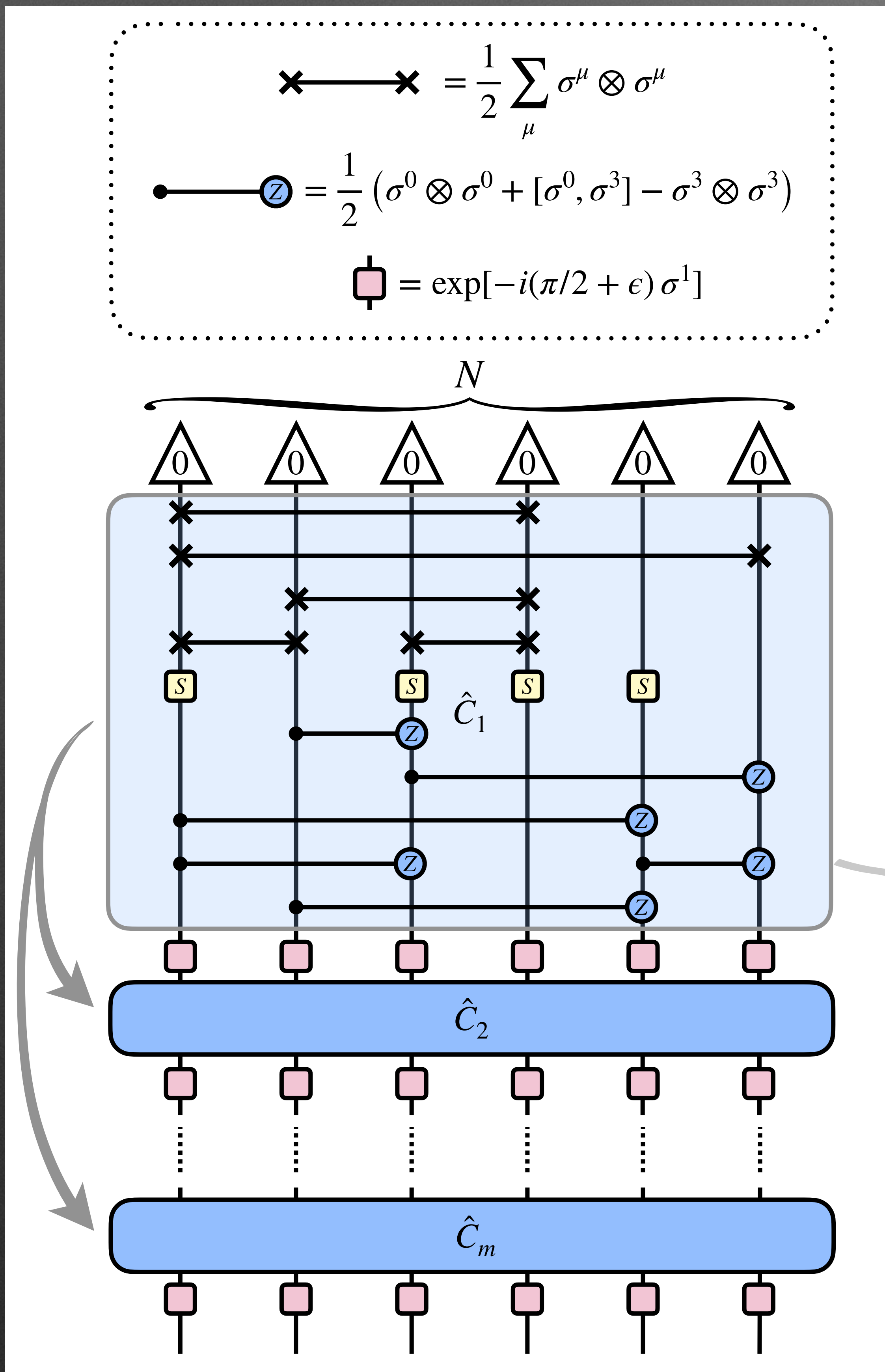
Entanglement growth

$$|\psi\rangle = C_2 T C_1 |\psi\rangle = C_2 C_1 \underbrace{\boxed{C_1^\dagger T C_1 |\psi\rangle}}_{|\tilde{\psi}\rangle}$$

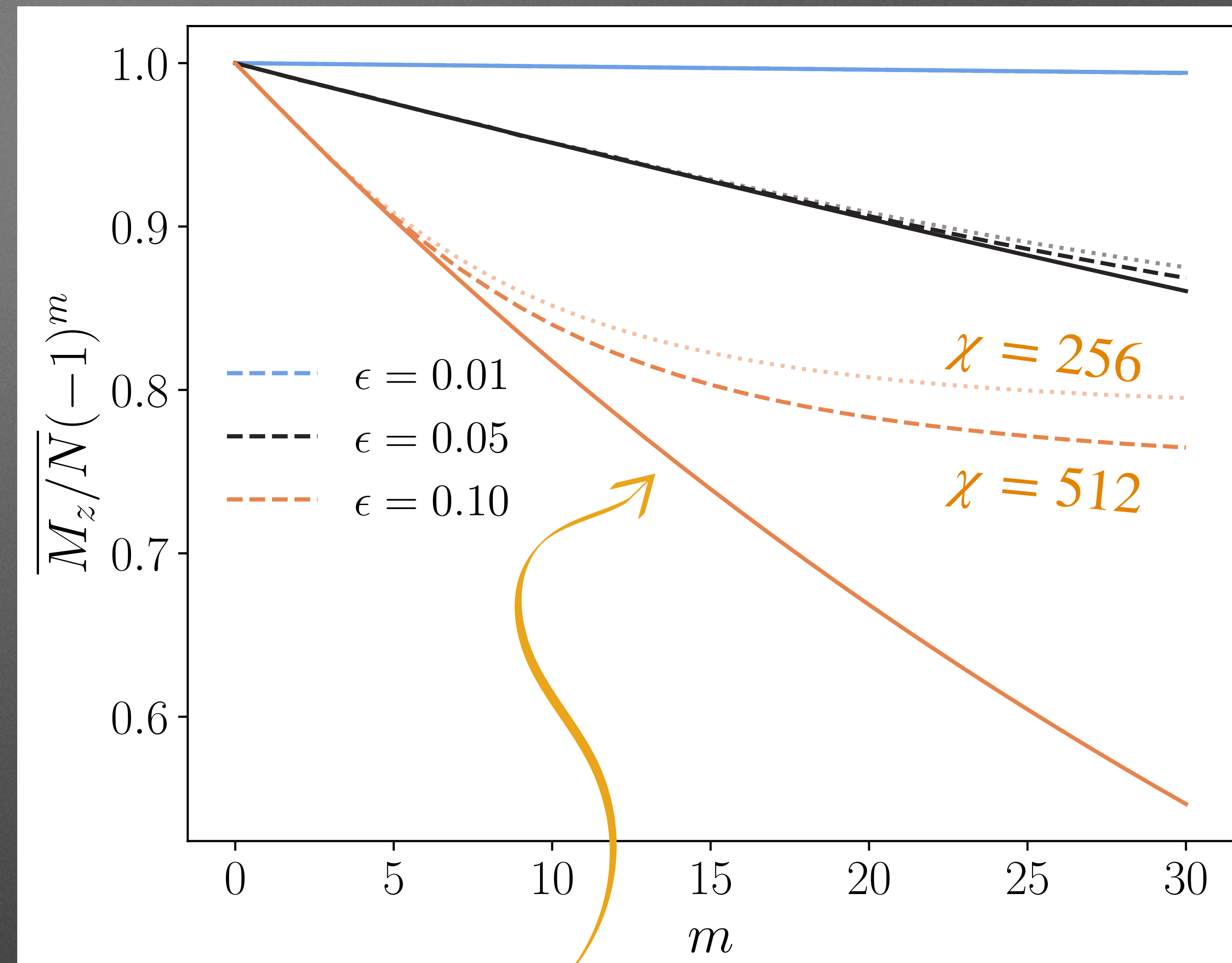
$N = 40$



Random Clifford–Floquet U(1) Kicked



$N = 40$ Magnetization

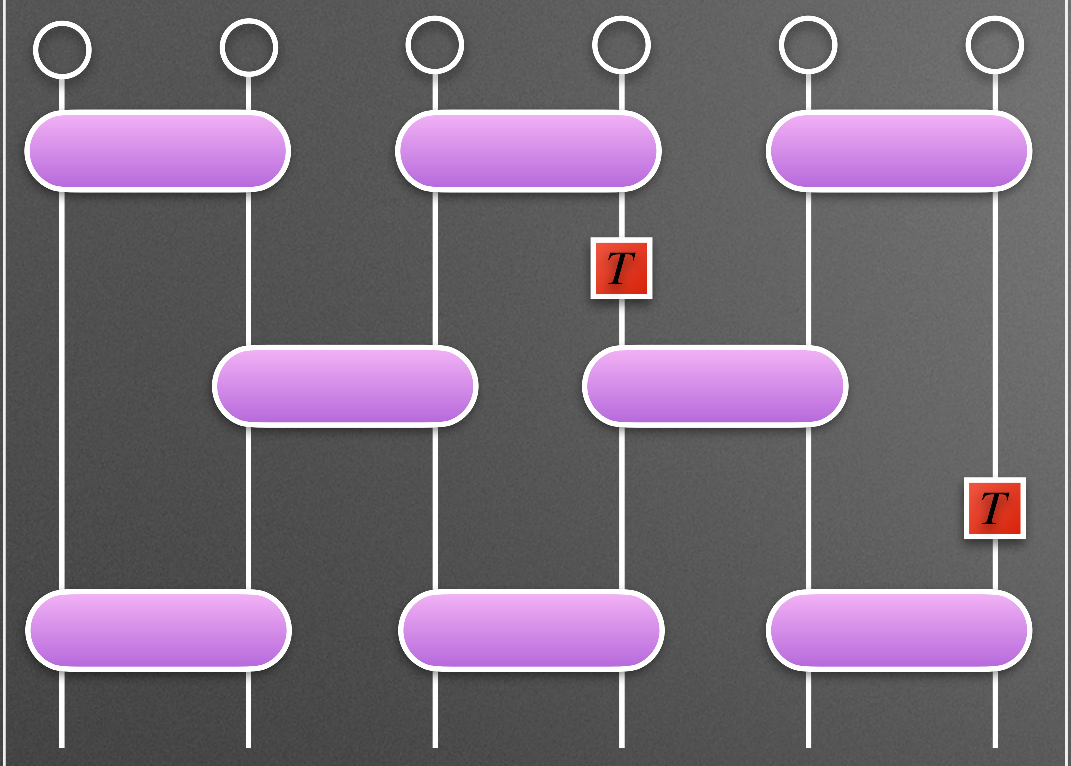


Analytic $\overline{\langle \sigma_j^z \rangle} = (-1)^m [\cos(2\epsilon)]^m$

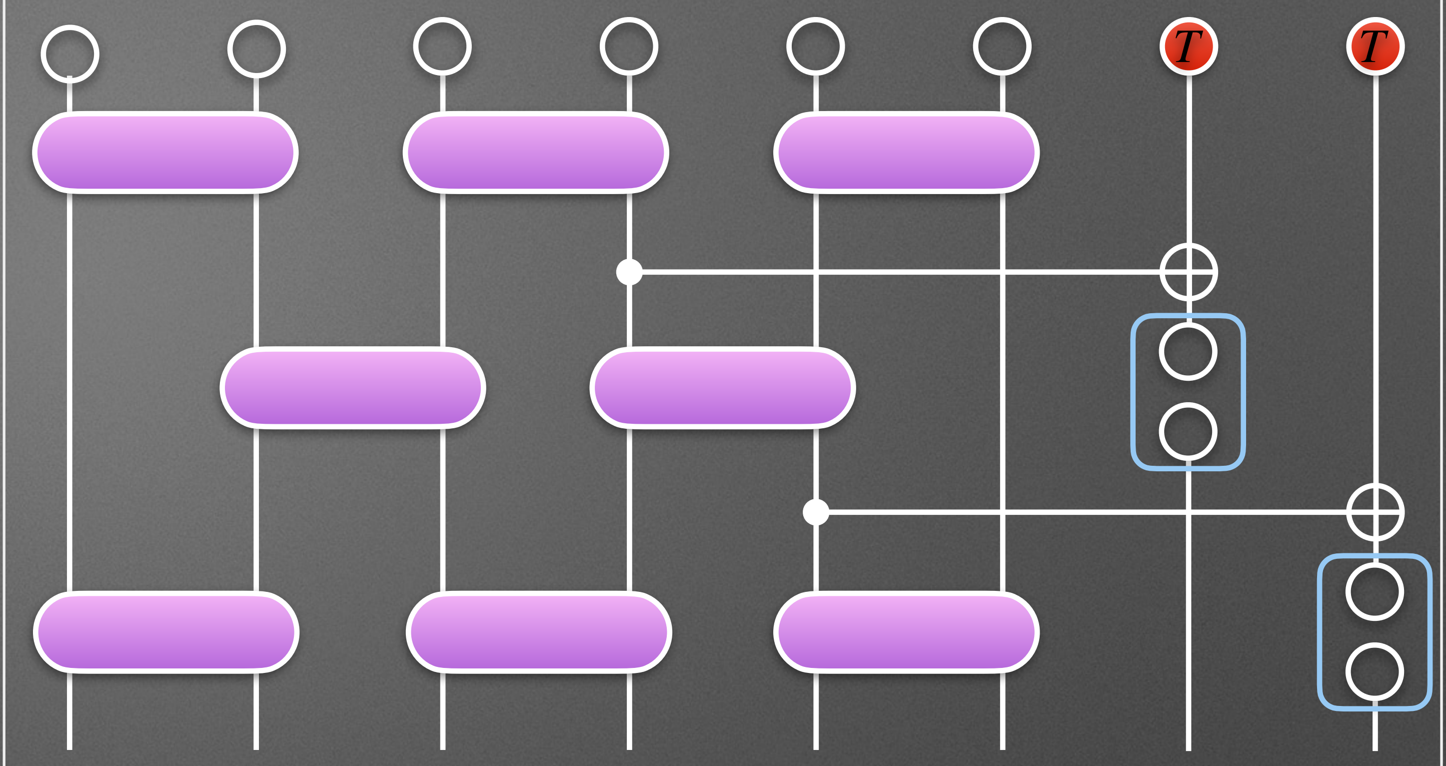
Random N qubits U(1) symmetric Clifford Unitary

$$\left(\prod_{1 \le i < j \le N} \hat{CZ}_{ij}^{\nu_{ij}} \right) \left(\prod_{j=1}^{N} \hat{S}_j^{\mu_j} \right) \hat{P}_n$$

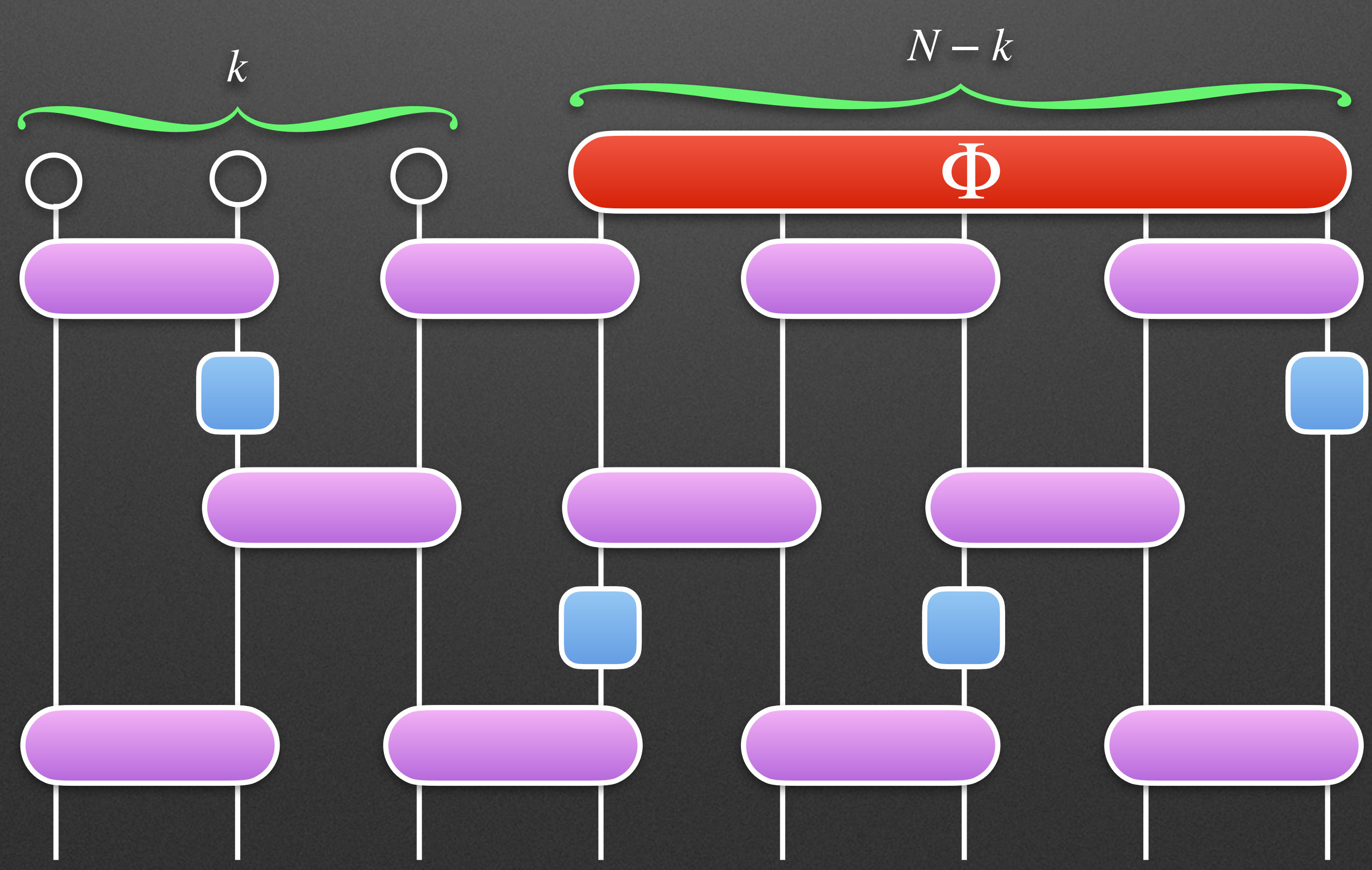
T-doped Circuit



T-Gadget & Measure

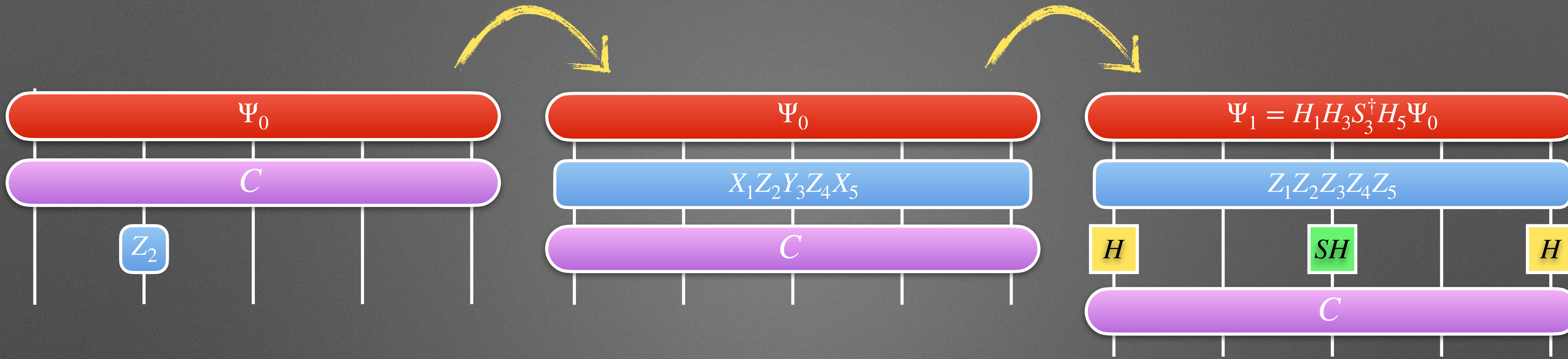


$$|\Psi_0\rangle = \bigotimes_{i=1}^{k} |\varphi_i\rangle \, |\Phi^{(N-k)}_{non-stab}\rangle$$

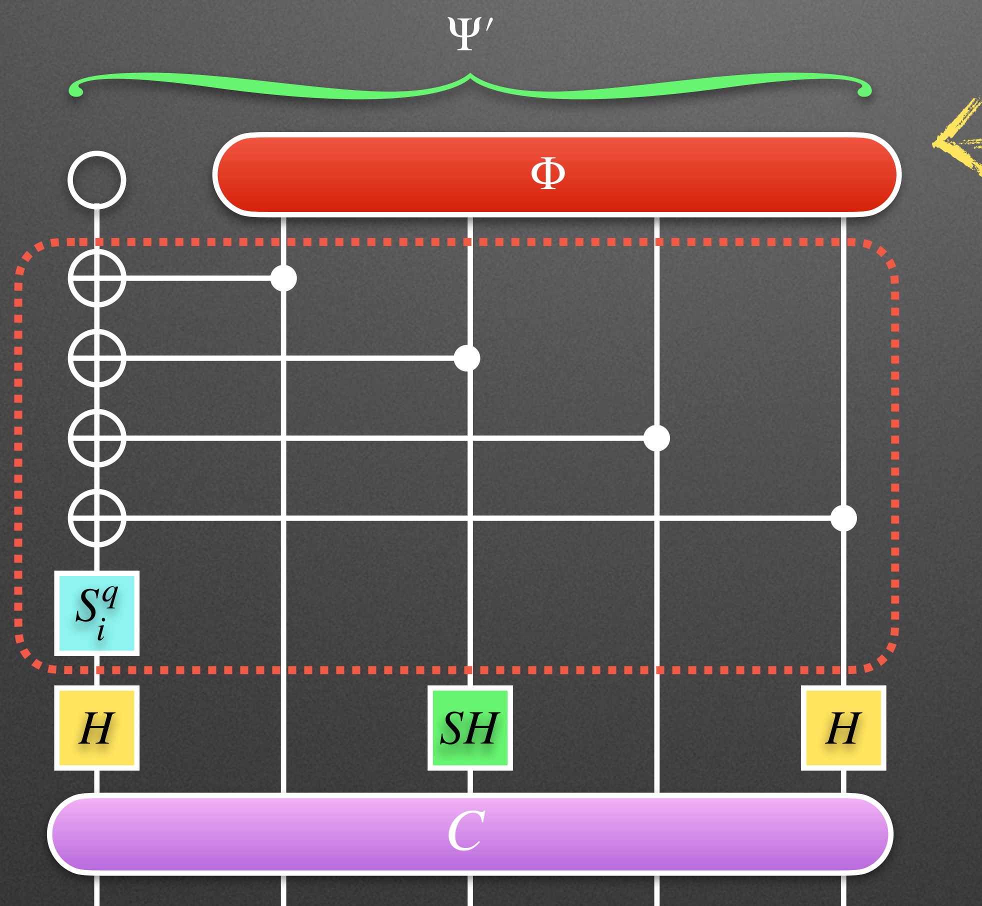


Measure & Clifford On NON-Stabiliser state

A Paviglianiti, G Lami, M Collura, A Silva, arXiv: 2405.06054



Replacing projector to Clifford unitaries enforcing partity constraint



Remember: $|\Psi_1\rangle = \bigotimes_{i=1}^{k} |\tilde{\varphi}_i\rangle \, |\tilde{\Phi}^{(N-k)}_{non-stab}\rangle$

$i \in \{1,\ldots,k\}$ Non-stabiliserness keeps unchanged

$$|\Psi'\rangle = \bigotimes_{j\neq i}^{k} |\tilde{\varphi}_j\rangle \, |0_i\rangle \, |\tilde{\Phi}^{(N-k)}_{non-stab}\rangle$$

$i \notin \{1,\ldots,k\}$ Non-stabiliserness does change

$$|\Psi'\rangle \approx \bigotimes_{j=1}^{k} |\tilde{\varphi}_j\rangle \, |0_i\rangle \, |\tilde{\Phi}^{(N-k-1)}_{non-stab}\rangle$$

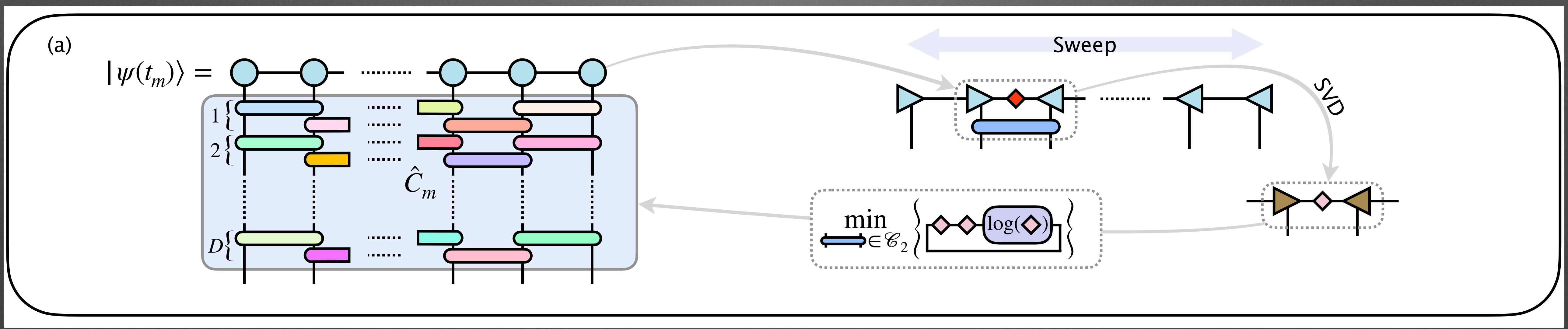
Variationally as an MPS for example

A Paviglianiti, G Lami, M Collura, A Silva, arXiv: 2405.06054

Clifford Dressed TDVP

A.F. Mello, A. Santini, G. Lami, J. De Nardis, M. Collura, arXiv:2407.01692

$$|\psi(t_m)\rangle = U(m\mathrm{d}t)\,|\psi_0\rangle \qquad\qquad |\tilde{\psi}(t_m)\rangle = C\,|\psi(t_m)\rangle$$



C is found variationally

720 unsigned Clifford unitaries

... actually the disentangling ones are O(10)

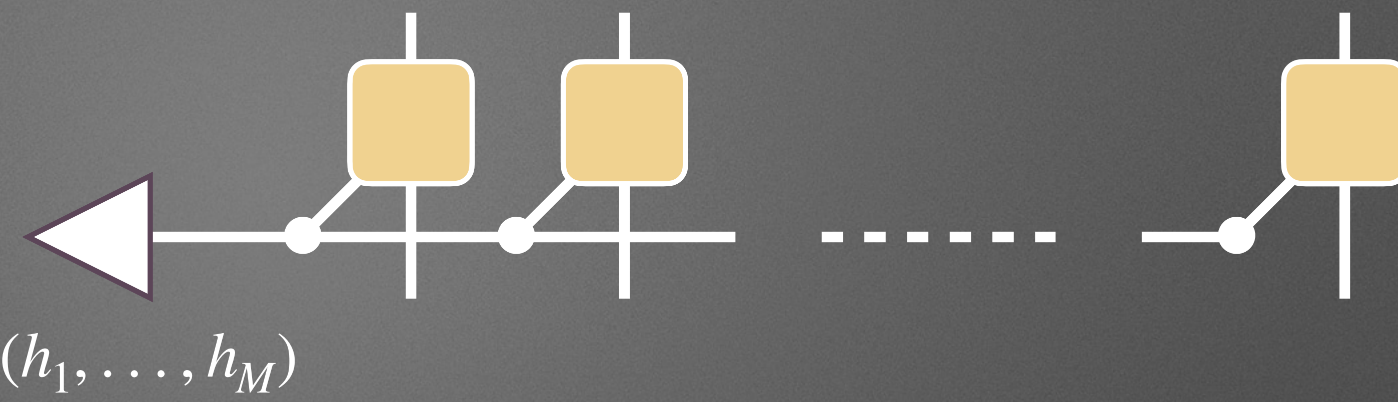
See also:

G. Lami, T. Haug, J. De Nardis, arXiv:2404.18751.

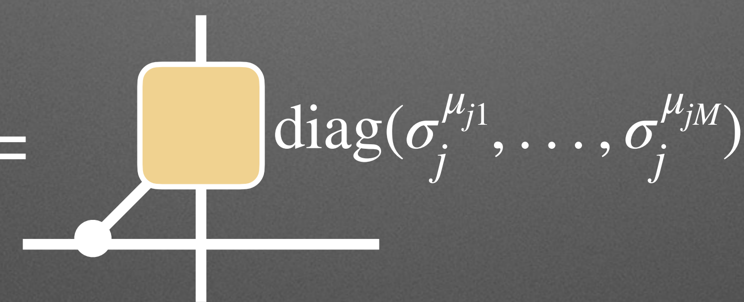
X. Qian, J. Huang, M. Qin, arXiv:2405.09217 and arXiv:2407.03202.

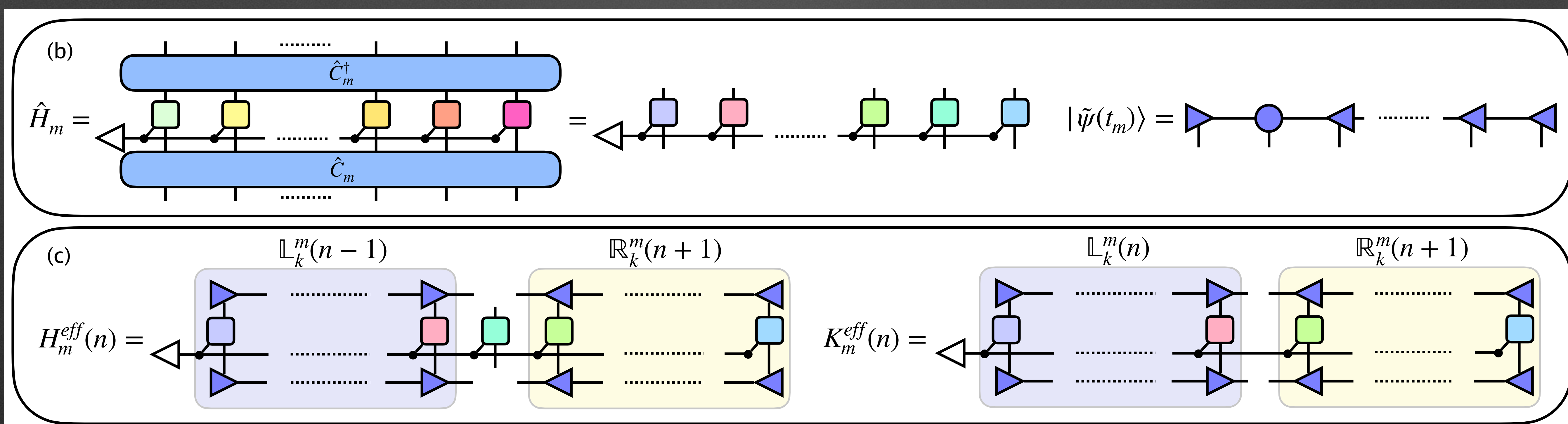
Clifford Dressed TDVP

$$H = \sum_{m=1}^{M} h_m \sigma_1^{\mu_{1m}} \ldots \sigma_N^{\mu_{Nm}}$$



$W_j =$



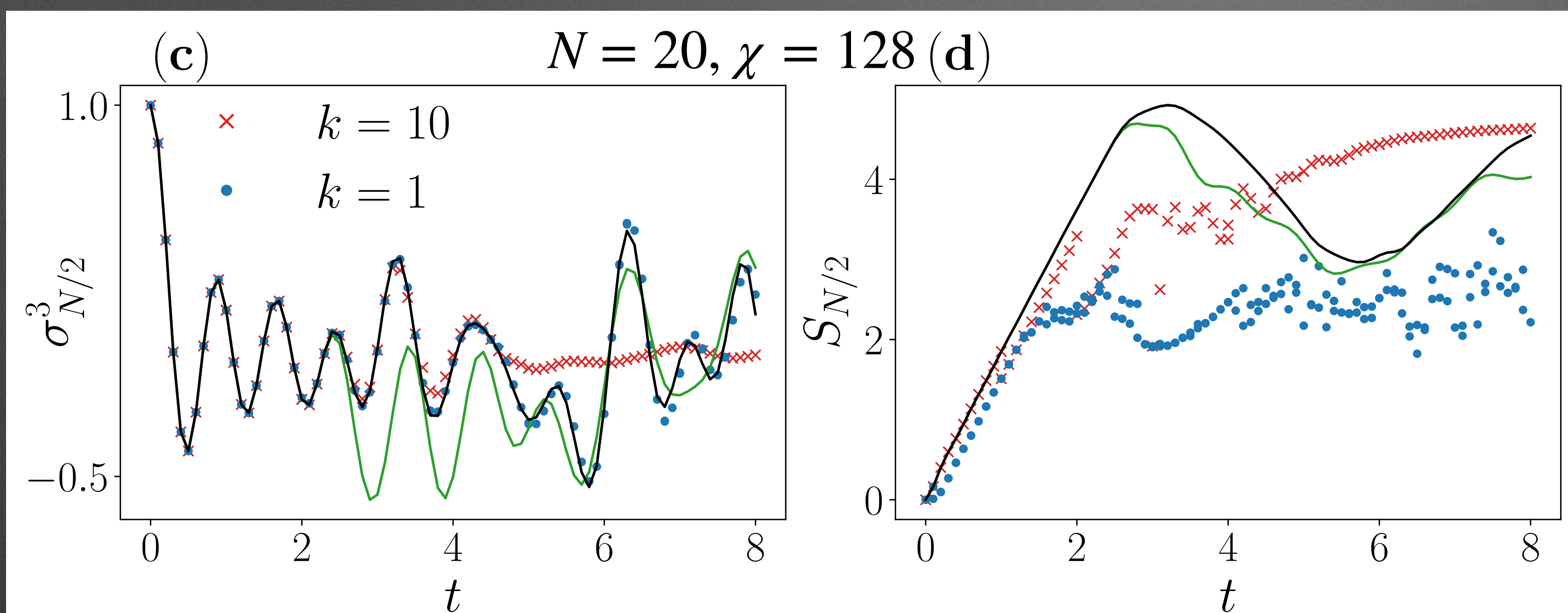


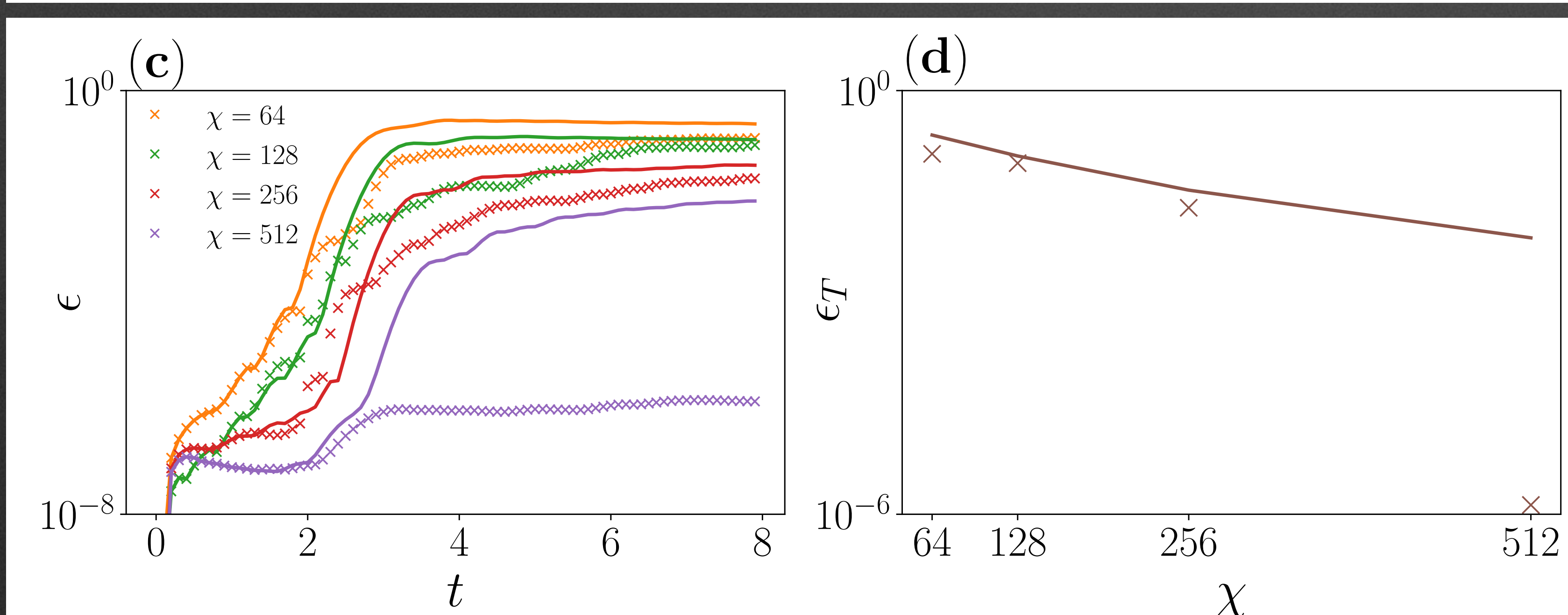
Clifford Dressed TDVP

XX model

$$H = \sum_j X_j X_{j+1} + Y_j Y_{j+1}$$

$$|\psi_0\rangle = |01,\dots,01\rangle$$





$$\varepsilon_t = \frac{1}{t}\int_0^t |\, O_{\mathrm{exact}}(t) - \tilde{O}_\chi(t)\,|$$

Take Home Messages

- Clifford formalism and Matrix Product States can be used together to simplify quantum simulations by reducing complexity.
- Up to a point in which entanglement and magic become inseparable.
- This point, determined by the specific quantum state or task, defines the threshold where classical methods fail, necessitating quantum computation.